Consider an n-state NFA with states $0, 1, \ldots, n-1$, the initial state 0, the accepting states $\{n-1\}$ and transitions on two symbols:
$a : 0 \to 1 \to \ldots \to n-1$,
$b : n-1 \to n-2 \to \ldots \to 0$.
Let L be the language accepted by this NFA.

For $k = 0, 1, \ldots, n-1$, consider the following sets of pairs of strings

$$\mathcal{A}_k = \{(b^k a^i, a^{2n-2-i} b^{n-1-k}) \mid i = 0, 1, \ldots, 2n-2\},$$

i.e., we have n sets each of which consists of $2n-1$ pairs.

We will show that the set $\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_1 \cup \ldots \cup \mathcal{A}_{n-1}$ is a fooling set for the language $shift(L)$. We need to show that
(1) for any pair (x_i, y_i) in $\mathcal{A}$, the string $x_i y_i$ is in the language $shift(L)$,
(2) for any two different pairs $(x_i, y_i), (x_j, y_j)$ in $\mathcal{A}$, **at least one** of the strings $x_i y_j$ and $x_j y_i$ is not in the language $shift(L)$.

To prove (1) note that for any k and any i, we have:

$$b^k a^i a^{2n-2-i} b^{n-1-k} = b^k a^{2n-2} b^{n-1-k},$$

which is in $shift(L)$ since the string $a^{n-1} b^{n-1-k} b^k a^{n-1}$ is in L.

To prove (2) we have two cases to consider:

(i) two different pairs in $\mathcal{A}_k$:
$(b^k a^i, a^{2n-2-i} b^{n-1-k})$ and
$(b^k a^j, a^{2n-2-j} b^{n-1-k})$,
where $0 \le i < j \le 2n-2$. Take the first part of the j-th pair and the second part of the i-th pair:

$$b^k a^j a^{2n-2-i} b^{n-1-k} = b^k a^{2n-2+j-i} b^{n-1-k},$$

where $2n-2+j-i > 2n-2$ which means that any cyclic shift of this string contains a string a^r with $r > n-1$ as a substring, and so is not in L. Thus, the string $b^k a^j a^{2n-2-i} b^{n-1-k}$ is not in $shift(L)$.

(ii) a pair $(b^k a^i, a^{2n-2-i} b^{n-1-k})$ in $\mathcal{A}_k$ and
a pair $(b^l a^j, a^{2n-2-j} b^{n-1-l})$ in $\mathcal{A}_l$,
where $0 \le k < l \le n-1$. Take the first part of the pair in $\mathcal{A}_l$ and the second part of the pair in $\mathcal{A}_k$ to get the string

$$b^l a^j a^{2n-2-i} b^{n-1-k}.$$

Note that any cyclic shift of this string ending in a contains a string $b^{n-1+l-k}$, where $n-1+l-k > n-1$, as a substring, and so is not in L. Thus, the string $b^l a^j a^{2n-2-i} b^{n-1-k}$ is not in $shift(L)$.

Hence the set $\mathcal{A}$ is a fooling set for the language $shift(L)$ of size $2n^2 - n$ which follows that any NFA for the language $shift(L)$ needs at least $2n^2 - n$ states.