State BL-algebras

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Outline

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- Basic definitions
- State BL-algebras, examples, properties
- Strong state BL-algebras
- State-morphism BL-algebras
- States on state BL-algebras
- Special classes of state BL-algebras
  - simple, semisimple, perfect, local

History

- T. Flaminio, F. Montagna - MV-algebras with an internal state - *state MV-algebras*
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- A. Di Nola, A. Dvurečenskij - *state-morphism MV-algebras*
- internal state as endomorphism
- J. Rachůnek, D. Šalounová - *state pseudo MV-algebras*
- a generalization for pseudo MV-algebras

Basic definitions - BL algebras

- P. Hájek - algebraic model of fuzzy logic of continuous t-norms
- A *BL-algebra* is an algebra $(A, \wedge, \vee, \odot, \rightarrow, 0, 1)$ of the type $(2, 2, 2, 2, 0, 0)$ such that
  - $(A, \wedge, \vee, 0, 1)$ is a bounded lattice,
  - $(A, \odot, 1)$ is a commutative monoid,
  - $c \leq a \rightarrow b$ iff $a \odot c \leq b$, (*adjointness*)
  - $a \wedge b = a \odot (a \rightarrow b)$, (*divisibility*)
  - $(a \rightarrow b) \vee (b \rightarrow a) = 1$, (*prelinearity*)

  for all a, b, c in A.

Basic definitions - states

- A *Bosbach state*, or a *state* on A is a function $s : A \longrightarrow [0,1]$ with the following properties:
  $(BS1)$ $s(x) + s(x \longrightarrow y) = s(y) + s(y \longrightarrow x)$;
  $(BS2)$ $s(0) = 0$ and $s(1) = 1$, for any x, y in A.

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- A function $s : A \longrightarrow [0,1]$ is called a *Riečan state* if the following conditions hold:
  $(RS1)$ if $x \perp y$, then $s(x + y) = s(x) + s(y)$;
  $(RS2)$ $s(0) = 0$,
  - $x \perp y$ denotes *orthogonal elements*, i. e. $x^{--} \leq y^{-}$.
  - For two orthogonal elements x, y we define $x + y := y^{-} \longrightarrow x^{--} (= x^{-} \longrightarrow y^{--})$.

Basic definitions

- A. Dvurečenskij, J. Rachůnek
  - states and Riečan states coincide on BL
- $x \oplus y := (x^- \odot y^-)^-$,
- $x \ominus y := x \odot y^-$,
- $d(x, y) = (x \to y) \odot (y \to x)$
- ord(x) - the smallest n such that $x^n = 0$.
  If such n does not exist, then ord$(x) = \infty$.
- Rad(A)
  - the intersection of all maximal filters in A

State BL-algebras

Let A be a BL-algebra and let $\sigma : A \to A$ be a mapping with the following properties:

1. $\sigma(0) = 0$;
2. $\sigma(x \to y) = \sigma(x) \to \sigma(x \wedge y)$;
3. $\sigma(x \odot y) = \sigma(x) \odot \sigma(x \to x \odot y)$;
4. $\sigma(\sigma(x) \odot \sigma(y)) = \sigma(x) \odot \sigma(y)$;
5. $\sigma(\sigma(x) \to \sigma(y)) = \sigma(x) \to \sigma(y)$.

Then σ is called a *state operator* and (A, σ) a *state-morphism BL-algebra*.

State BL-algebras - examples

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- E. Turunen - Consider $A = \{0, a, b, 1\}$ where $0 < a < b < 1$. Then $(A, \wedge, \vee, \odot, \rightarrow, 0, 1)$ is a BL-algebra with the operations:

$\odot$	**0**	**a**	**b**	**1**
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
1	0	a	b	1

$\rightarrow$	**0**	**a**	**b**	**1**
0	1	1	1	1
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$\rightarrow$	**0**	**a**	**b**	**1**
0	1	1	1	1
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- $\sigma(0) = 0, \sigma(a) = a, \sigma(b) = 1, \sigma(1) = 1$
- (A, σ) is a state BL-algebra

State BL-algebras - examples

- Moreover, $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$ and
- $\sigma(x \to y) = \sigma(x) \to \sigma(y)$.

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  - $\sigma(x \to y) = \sigma(x) \to \sigma(y)$.
- A. Di Nola, A. Dvurečenskij for MV-algebras
- Let (A, σ) be a state BL-algebra. Let us consider $\sigma_1 : A \times A \to A \times A$ and $\sigma_2 : A \times A \to A \times A$, such that $\sigma_1((a, b)) = (a, a)$, $\sigma_2((a, b)) = (b, b)$. Then
  - $(A \times A, \sigma_1)$, $(A \times A, \sigma_2)$ - state BL-algebras.
  - $(A \times A, \sigma_1) \cong (A \times A, \sigma_2)$,
  - non-linear examples of subdirectly irreducible state BL-algebras
    (if A is subdirectly irreducible).

State BL-algebras - properties

- $\sigma(1) = 1$;
- $\sigma(x^-) = \sigma(x)^-$;
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  if $x \odot y = 0$ then $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$;

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- $\sigma(x \wedge y) = \sigma(x) \odot \sigma(x \to y)$;

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- $\sigma(x \wedge y) = \sigma(x) \odot \sigma(x \to y)$;
- $\sigma(x \to y) \leq \sigma(x) \to \sigma(y)$ and if x, y are comparable then $\sigma(x \to y) = \sigma(x) \to \sigma(y)$;

Properties

- $\sigma(x \to y) \odot \sigma(y \to x) \leq d(\sigma(x), \sigma(y));$

Properties

- $\sigma(x \to y) \odot \sigma(y \to x) \leq d(\sigma(x), \sigma(y))$;
- $\sigma(x) \oplus \sigma(y) \geq \sigma(x \oplus y)$ and if $x \oplus y = 1$ then $\sigma(x) \oplus \sigma(y) = \sigma(x \oplus y) = 1$;

Properties

- $\sigma(x \to y) \odot \sigma(y \to x) \leq d(\sigma(x), \sigma(y))$;
- $\sigma(x) \oplus \sigma(y) \geq \sigma(x \oplus y)$ and if $x \oplus y = 1$ then $\sigma(x) \oplus \sigma(y) = \sigma(x \oplus y) = 1$;
- $\sigma(\sigma(x)) = \sigma(x)$;
- $\sigma(A)$ is a BL-subalgebra of A;
- $\sigma(A) = \{x \in A : x = \sigma(x)\}$;
- if $ord(x) < \infty$, then $\sigma(x) \notin Rad(A)$.

Properties

- $\sigma(x \to y) = \sigma(x) \to \sigma(y) \Leftrightarrow \sigma(x \wedge y) = \sigma(x) \wedge \sigma(y)$.

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- The operator σ preserves the operation $\to$ (as homomorphism) iff σ preserves the operation $\wedge$.

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- The operator σ preserves the operation $\to$ (as homomorphism) iff σ preserves the operation $\wedge$.
- σ preserves $\to$ iff σ preserves $\vee$.

Properties

- $\sigma(x \rightarrow y) = \sigma(x) \rightarrow \sigma(y) \Leftrightarrow$
  $\sigma(x \wedge y) = \sigma(x) \wedge \sigma(y)$.
- The operator σ preserves the operation $\rightarrow$ (as homomorphism) iff σ preserves the operation $\wedge$.
- σ preserves $\rightarrow$ iff σ preserves $\vee$.
- $x \vee y = [(x \rightarrow y) \rightarrow y] \wedge [(y \rightarrow x) \rightarrow x]$

Properties

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  $\sigma(x \wedge y) = \sigma(x) \wedge \sigma(y)$.
- The operator σ preserves the operation $\to$ (as homomorphism) iff σ preserves the operation $\wedge$.
- σ preserves $\to$ iff σ preserves $\vee$.
- $x \vee y = [(x \to y) \to y] \wedge [(y \to x) \to x]$
- $(x \vee y) \to y = x \to y$

Strong state BL-algebras

- Cignoli, Esteva, Godo, Torrens
  - for MV-algebras and linear product
  BL-algebras:

$$x \rightarrow x \odot y = x^{-} \vee y$$

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  $$x \longrightarrow x \odot y = x^- \vee y$$
- *Strong state BL-algebra* - axiom 3. is replaced by
  3'. $\sigma(x \odot y) = \sigma(x) \odot \sigma(x^- \vee y)$

Strong state BL-algebras

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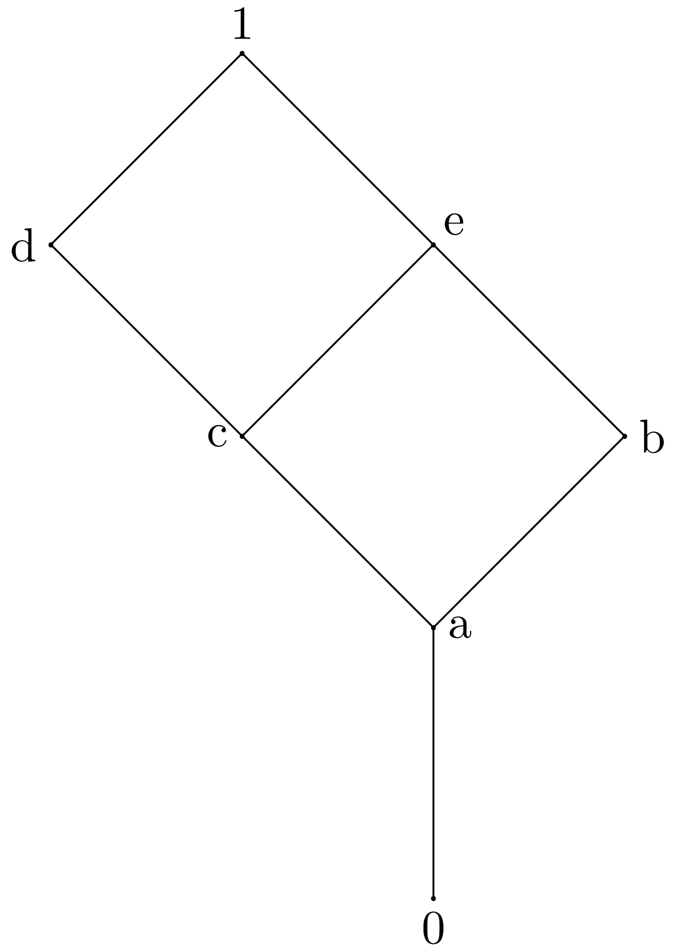
  BL-algebras:
  $$x \to x \odot y = x^- \vee y$$
- *Strong state BL-algebra* - axiom 3. is replaced by
  3'. $\sigma(x \odot y) = \sigma(x) \odot \sigma(x^- \vee y)$
- every strong state BL-algebra is a state BL-algebra
- the converse is not true

Counterexample

Consider the following state BL-algebra $A = \{0, a, b, c, d, e, 1\}$, with the order:



Counterexample

The operations are given by the tables:

$\odot$	**0**	**a**	**b**	**c**	**d**	**e**	**1**
0	0	0	0	0	0	0	0
a	0	0	a	0	0	a	a
b	0	a	b	a	a	b	b
c	0	0	a	0	c	a	c
d	0	0	a	c	d	c	d
e	0	a	b	a	c	b	e
1	0	a	b	c	d	e	1

Counterexample

$\rightarrow$	**0**	**a**	**b**	**c**	**d**	**e**	**1**
0	1	1	1	1	1	1	1
a	d	1	1	1	1	1	1
b	0	d	1	d	d	1	1
c	c	e	e	1	1	1	1
d	a	b	b	e	1	e	1
e	0	c	e	d	d	1	1
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Counterexample

$\to$	**0**	**a**	**b**	**c**	**d**	**e**	**1**
0	1	1	1	1	1	1	1
a	d	1	1	1	1	1	1
b	0	d	1	d	d	1	1
c	c	e	e	1	1	1	1
d	a	b	b	e	1	e	1
e	0	c	e	d	d	1	1
1	0	a	b	c	d	e	1

	0	**a**	**b**	**c**	**d**	**e**	**1**
σ	0	c	1	c	c	1	1

Counterexample

- Then (A, σ) is a state BL-algebra, but axiom 3'. fails for the pairs $(x, y) \in \{(c, d), (d, c), (d, d)\}$.
- It holds $\sigma(x \to y) = \sigma(x) \to \sigma(y)$, but $\sigma(x \odot y) \neq \sigma(x) \odot \sigma(y)$, e.g. for (d, d).

Properties - (strong) state BL

- $\sigma(x \odot y) \geq \sigma(x) \odot \sigma(y)$ and if $x^- \leq y$ then $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$;
- $\sigma(x \ominus y) \geq \sigma(x) \ominus \sigma(y)$ and if x and y are comparable then $\sigma(x \ominus y) = \sigma(x) \ominus \sigma(y)$;

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- **Lemma:** *Let (A, σ) be a linearly ordered state BL-algebra. Then for $x, y \in A$ we have:*
  (1) $\sigma(x \to y) = \sigma(x) \to \sigma(y)$;
  Moreover if (A, σ) is strong, we have:
  (2) $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$.

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- **Lemma:** *Let* (A, σ) *be a linearly ordered state BL-algebra. Then for* $x, y \in A$ *we have:*
  (1) $\sigma(x \to y) = \sigma(x) \to \sigma(y)$*;*
  Moreover if (A, σ) *is strong, we have:*
  (2) $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$.
- State BL-algebras on the subclass of MV-algebras ($x^{--} = x$) coincide with state MV-algebras defined by FlMo.

State-morphism BL-algebras

- Let (A, σ) be a state BL-algebra. If the operator σ satisfies the following properties:
  - $\sigma(x \odot y) = \sigma(x) \odot \sigma(y)$,
  - $\sigma(x \to y) = \sigma(x) \to \sigma(y)$,

  for $x, y \in A$. Then σ is called a *state-morphism operator* on A and (A, σ) is a *state-morphism BL-algebra*.

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  for $x, y \in A$. Then σ is called a *state-morphism operator* on A and (A, σ) is a *state-morphism BL-algebra*.
- "Turunen's example" is a state-morphism BL-algebra, the 7-element example is not.
- Every linearly ordered strong state BL-algebra is a state-morphism BL-algebra.

States

- Let (A, σ) be a state BL-algebra, and let s be a state on A. Then $s_\sigma(x) := s(\sigma(x))$ for x in A is a state on A.

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- *σ-compatible state*, if $\sigma(x) = \sigma(y)$ then $s(x) = s(y)$, for a state s.

States

- Let (A, σ) be a state BL-algebra, and let s be a state on A. Then $s_\sigma(x) := s(\sigma(x))$ for x in A is a state on A.
- *σ-compatible state*, if $\sigma(x) = \sigma(y)$ then $s(x) = s(y)$, for a state s.
- **Theorem:** *Let (A, σ) be a state BL-algebra. Then there is a bijective correspondence between the σ-compatible states on A and the states on $\sigma(A)$.*

Special classes

- **Def:** Let (A, σ) be a state BL-algebra (or a state-morphism BL-algebra). A nonempty set $F \subseteq A$ is called a *state-filter* (or a *state-morphism filter*) of A if F is a filter of A such that if $x \in F$, then $\sigma(x) \in F$.

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- Let (A, σ) be a state-morphism BL-algebra. Then
$$\sigma(Rad(A)) = Rad(\sigma(A)).$$

Special classes

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- Let (A, σ) be a state-morphism BL-algebra. Then
$$\sigma(Rad(A)) = Rad(\sigma(A)).$$
- For state BL-algebras we have
$$Rad(\sigma(A)) \subseteq \sigma(Rad(A)).$$

Subclasses - simple

- Recall: A BL-algebra A is called *simple* if A has two filters.

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- **Def:** A state BL-algebra (A, σ) is called *simple* if $\sigma(A)$ is simple. We denote by $SSBL$ the class of all simple state BL-algebras.

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- (A, σ) a state BL-algebra. If A is simple as BL-algebra then (A, σ) is a simple state BL-algebra.

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- (A, σ) a state BL-algebra. If A is simple as BL-algebra then (A, σ) is a simple state BL-algebra.
- **Thm:** Let (A, σ) be a state-morphism BL-algebra. The following are equivalent:
  (1) $(A, \sigma) \in SSBL$;
  (2) $\ker(\sigma)$ is a maximal filter of A.

Subclasses - semisimple

- Recall: A is *semisimple* if $Rad(A) = \{1\}$.

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- **Def:** A state BL-algebra (A, σ) is called *semisimple* if $Rad(\sigma(A)) = \{1\}$. We denote by $SSSBL$ the class of all semisimple state BL-algebras.

Subclasses - semisimple

- Recall: A is *semisimple* if $Rad(A) = \{1\}$.
- **Def:** A state BL-algebra (A, σ) is called *semisimple* if $Rad(\sigma(A)) = \{1\}$. We denote by $SSSBL$ the class of all semisimple state BL-algebras.
- (A, σ) a state BL-algebra. If A is a semisimple BL-algebra then (A, σ) is a semisimple state BL-algebra.

Subclasses - semisimple

- Recall: A is *semisimple* if $Rad(A) = \{1\}$.
- **Def:** A state BL-algebra (A, σ) is called *semisimple* if $Rad(\sigma(A)) = \{1\}$. We denote by $SSSBL$ the class of all semisimple state BL-algebras.
- (A, σ) a state BL-algebra. If A is a semisimple BL-algebra then (A, σ) is a semisimple state BL-algebra.
- **Thm:** Let (A, σ) be a state-morphism BL-algebra. The following are equivalent:
  (1) $(A, \sigma) \in SSSBL$;
  (2) $Rad(A) \subseteq \ker(\sigma)$.

Subclasses - perfect

- Recall: A BL-algebra is called *perfect* if $x \in Rad(A)$ or $x \in Rad(A)^-$, for any $x \in A$, where $Rad(A)^- = \{x^- : x \in Rad(A)\}$.

Subclasses - perfect

- Recall: A BL-algebra is called *perfect* if $x \in Rad(A)$ or $x \in Rad(A)^-$, for any $x \in A$, where $Rad(A)^- = \{x^- : x \in Rad(A)\}$.
- **Thm:** Let (A, σ) be a state BL-algebra. The following are equivalent:
  (1) A is perfect;
  (2) $(\forall x \in A, \sigma(x) \in Rad(A)$ implies $x \in Rad(A))$ and $\sigma(A)$ is perfect.

Subclasses - local

- **Def:** Let (A, σ) be a state BL-algebra. σ is called *radical-faithful* if, for every $x \in A$, $\sigma(x) \in Rad(A)$ implies $x \in Rad(A)$.

Subclasses - local

- **Def:** Let (A, σ) be a state BL-algebra. σ is called *radical-faithful* if, for every $x \in A$, $\sigma(x) \in Rad(A)$ implies $x \in Rad(A)$.
- Recall: A BL-algebra is called *local* if it has a unique maximal filter.

Subclasses - local

- **Def:** Let (A, σ) be a state BL-algebra. σ is called *radical-faithful* if, for every $x \in A$, $\sigma(x) \in Rad(A)$ implies $x \in Rad(A)$.
- Recall: A BL-algebra is called *local* if it has a unique maximal filter.
- **Thm:** Let (A, σ) be a radical-faithful state-morphism BL-algebra. The following are equivalent:
  (1) A is a local BL-algebra;
  (2) $\sigma(A)$ is a local BL-algebra.

Subclasses - local

- **Thm:** Let (A, σ) be a state-morphism BL-algebra. The following are equivalent:
  (1) $(A, \sigma) \in SSBL$;
  (2) A is a local BL-algebra and
  $$\ker(\sigma) = Rad(A).$$

Subclasses - local

- **Thm:** Let (A, σ) be a state-morphism BL-algebra. The following are equivalent:
  (1) $(A, \sigma) \in SSBL$;
  (2) A is a local BL-algebra and
  $$\ker(\sigma) = Rad(A).$$
- Let B be a subalgebra of a BL-algebra A and let σ be a state operator of A. If $\sigma(B) \subseteq B$, then $\sigma(B)$ is a subalgebra of B.

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