Complexity of languages resulting from the cut operation in the unary case

Michal Hospodár

Mathematical Institute
Slovak Academy of Sciences
Košice, Slovakia

Joint work with Markus Holzer

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Basic Notions

Cut Operation

- Concatenation: $KL = \{uv \mid u \in K \text{ and } v \in L\}$
- Cut operation: machine implementation on Unix processors
  $K \,!\, L = \{\, uv \mid$
  $u \in K,\ v \in L, \text{ and } uv' \notin K \text{ for every nonempty prefix } v' \text{ of } v \,\}$

State Complexity

- State complexity of a regular language L:
  value $\mathrm{sc}(L) = \min\{n \mid L \text{ is accepted by a } \mathrm{DFA} \text{ with } n \text{ states}\}$
- State complexity of a binary operation $\circ$:
  function $n \mapsto \max\{\mathrm{sc}(K \circ L) \mid \mathrm{sc}(K) \leq m \text{ and } \mathrm{sc}(L) \leq n\}$
- Range of state complexities resulting from the operation $\circ$:
  set $\{\mathrm{sc}(K \circ L) \mid \mathrm{sc}(K) = m \text{ and } \mathrm{sc}(L) = n\}$

A number representing a "hole" in this set is called
a magic number for the operation $\circ$

Motivation and History

Cut operation was examined by

- Berglund et al. (2013) – definition, regularity preserving
- Drewes et al. (2017) – state complexity

Magic number problem was investigated by

- Iwama et al. (2000): determinization of binary NFAs
- Van Zijl (2005): determinization of unary XNFAs
- Geffert (2007): determinization of unary NFAs
- Holzer et al. (2012): determinization on subregular classes
- Čevorová (2013): Kleene star on unary DFAs
- ...

This paper – complexity of languages resulting from cut operation (the magic number problem for cut on unary languages)

Range of Complexities for the Cut Operation

Known result: Drewes, Holzer, Jakobi, van der Merwe (2017)

The state complexity of the cut operation on unary languages:

$$f(m,n) = \begin{cases} 1, & \text{if } m = 1; \\ m, & \text{if } m \geq 2 \text{ and } n = 1; \\ 2m - 1, & \text{if } m, n \geq 2 \text{ and } m \geq n; \\ m + n - 2, & \text{if } m, n \geq 2 \text{ and } m < n. \end{cases}$$

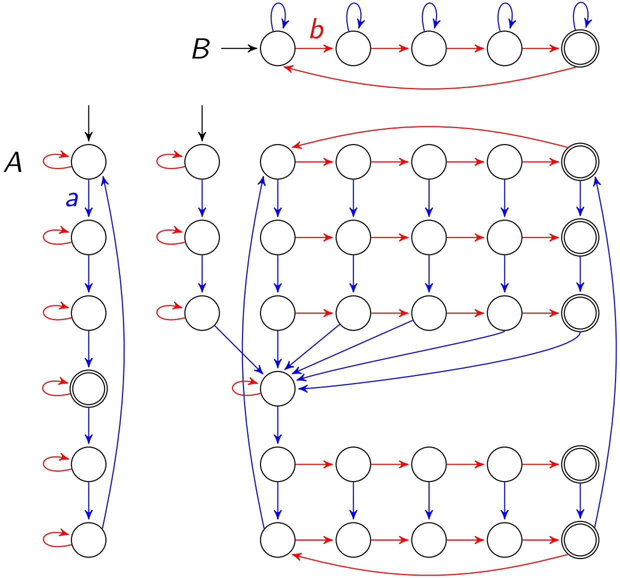
Results of this paper:

Let K, L be unary languages with $\mathrm{sc}(K) = m$ and $\mathrm{sc}(L) = n$.

Condition	Range of attainable complexities for $K ! L$
$m \geq 1, n = 1$	$[1, m]$
$m, n \geq 2$, K infinite	$[1, 2m - 1]$
$m, n \geq 2$, K finite	$[n, m + n - 2]$

- What about the interval $[2m, n - 1]$?

The Construction of the Cut Automaton



The Values from $2m$ up to $n-1$ Are Not Attainable

Theorem

There *do not exist* *minimal unary m- and n-state* DFAs *A and B such that the minimal* DFA *for $L(A)\,!\,L(B)$ has α states if $\alpha \in [2m, n-1]$.*

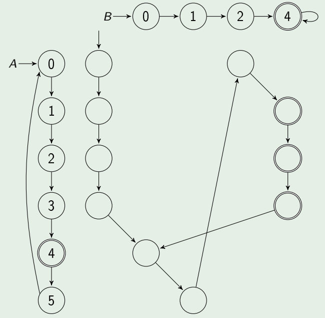
Proof

If $L(A)$ is infinite

- at most $m-1$ states are out of the product part
- DFA A has only one loop
  $\Rightarrow$ at most m states are in the product part

$\Rightarrow$ $\leq 2m-1$ reachable states

Example



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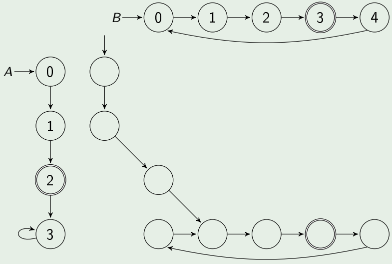

Proof (cont.)

If $L(A)$ is finite

- DFA A has a final state before its sink state
- in the product part, there is a copy of B

$\Rightarrow$ $\geq n$ reachable and distinguishable states □

Summary and Future Work

Condition	Range of attainable complexities for cut
$m = 1$	$\{1\}$
$n = 1$	$[1, m]$
$m, n \geq 2$	$[1, 2m - 1] \cup [n, m + n - 2]$

- if numbers from $2m$ up to $n - 1$ exist, they are not attainable (are magic)
- for every number from 1 to $f(m, n)$,
  we know whether it is or is not attainable
  $\Rightarrow$ the problem is completely solved for unary languages
- we do not know other operation where magic number problem is completely solved and magic numbers exist

Future work: magic number problem for cut on binary languages

- larger upper bound: $(m - 1)n + m$
- values in $[2m, n - 1]$ might not be magic in the binary case

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- DCFS 2019, July 17-19, Košice
- CIAA 2019, July 22-25, Košice