A Jumping $5' \rightarrow 3'$ WK Automata Model

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Jumping Finite Automata

(General) Jumping Finite Automaton (JFA)

$$\textit{quintuple} \quad M = (Q, \Sigma, R, s, F)$$

Q is a finite set of states

Σ is an input alphabet, $Q \cap \Sigma = \emptyset$

R is a finite set of rules: (p, y, q), where $p, q \in Q$, $y \in \Sigma^*$

s is the start state

F is a set of final states

Step/Move/Jump

- FA: $pyx \Rightarrow qx$ only if $(p, y, q) \in R$, $x \in \Sigma^*$
- JFA: $xpyz \curvearrowright x'qz'$ only if $(p, y, q) \in R$, $x, z, x', z' \in \Sigma^*$, $xz = x'z'$

Jumping Finite Automata

Example automaton

$$M = (\{s, p, q\}, \{a, b, c\}, R, s, \{s\})$$

where R:

$$(s, a, p)$$
$$(p, b, q)$$
$$(q, c, s)$$

Resulting language

- FA: $L(M) = \{(abc)^n : n \geq 0\}$
- JFA: $L(M) = \{w : w \in \{a, b, c\}^*,\ |w|_a = |w|_b = |w|_c\}$

Jumping Finite Automata – Extended Models

n-parallel jumping finite automata

- have *n* heads
- heads cannot cross each other
- in the right-jumping mode the behavior resembles:
  n-parallel right linear grammars, simple matrix grammars

Double-jumping finite automata

- always 2 heads
- heads cannot cross each other
- each had has its own restricted direction
- in some modes the model accepts only a subset of linear languages

Watson-Crick Automata

Watson-Crick finite automata

- biology-inspired model
- the core model is similar to FA
- work with the Watson-Crick tape
- uses two heads (one for each strand of the tape)

Watson-Crick tape

- double-stranded tape
- resembles DNA
- satisfies Watson-Crick complementary relation:
  the elements of the strands are pairwise complements of each other
  (e.g. (T, A), (A, T), (C, G), (G, C))

Watson-Crick Automata

$5' \to 3'$ Watson-Crick finite automata

- the heads read in the biochemical $5' \to 3'$ direction
- that is physically/mathematically in opposite directions

Sensing $5' \to 3'$ Watson-Crick finite automata

- the heads sense that they are meeting
- the processing of the input ends if for all pairs of the sequence one of the letters is read
  (due to the complementary relation, the sequence is fully processed)
- the tape notation is usually simplified: $\left[\frac{A}{T}\right]$ as a, ...

Sensing $5' \to 3'$ Watson-Crick Automata

Example steps

start: $\left[\begin{smallmatrix} A & A & T & C & G & A & C & T \\ T & T & A & G & C & T & G & A \end{smallmatrix}\right]$

1st step: $\left[\begin{smallmatrix} \color{red}{A} & A & T & C & G & A & C & T \\ T & T & A & G & C & T & G & A \end{smallmatrix}\right]$

2nd step: $\left[\begin{smallmatrix} \color{red}{A} & A & T & C & G & A & C & T \\ T & T & A & G & C & T & \color{blue}{G} & \color{blue}{A} \end{smallmatrix}\right]$

3rd step: $\left[\begin{smallmatrix} \color{red}{A} & \color{red}{A} & T & C & G & A & C & T \\ T & T & A & G & C & \color{blue}{T} & \color{blue}{G} & \color{blue}{A} \end{smallmatrix}\right]$

$\vdots$

last step: $\left[\begin{smallmatrix} \color{red}{A} & \color{red}{A} & \color{red}{T} & C & G & A & C & T \\ T & T & A & \color{blue}{G} & \color{blue}{C} & \color{blue}{T} & \color{blue}{G} & \color{blue}{A} \end{smallmatrix}\right]$

Accepting power

the family of linear languages

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Combined idea

Combined model

- the combination of (G)JFA and sensing $5' \to 3'$ WKA
- two heads as in sensing $5' \to 3'$ WKA
- each head can traverse the whole input in its direction
- all pairs of symbols are read only once

Expectations

- better accepting power than the non-combined models
- ability to model languages with some crossed agreements

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Final definition

Jumping $5' \to 3'$ WK automaton

$$\textit{quintuple} \quad M = (V, Q, q_0, F, \delta)$$

V, Q, q_0, F as in FA, $V \cap \{\#\} = \emptyset$,
$\delta \colon (Q \times V^* \times V^* \times D) \to 2^Q$ (finite),
$D = \{\oplus, \ominus\}$ indicates the mutual position of heads.

Configuration

$$(q, s, w_1, w_2, w_3)$$

- q is the state
- s is the position of heads
- w_1 is the unprocessed input before the first head
- w_2 is the unprocessed input between the heads
- w_3 is the unprocessed input after the second head

Final definition

Steps

Let $x, y, u, v, w_2 \in V^*$ and $w_1, w_3 \in (V \cup \{\#\})^*$.

1. $\oplus$-reading: $(q, \oplus, w_1, xw_2y, w_3) \curvearrowright (q', s, w_1\{\#\}^{|x|}, w_2, \{\#\}^{|y|}w_3)$, where $q' \in \delta(q, x, y, \oplus)$, and s is either $\oplus$ if $|w_2| > 0$ or $\ominus$.
2. $\ominus$-reading: $(q, \ominus, w_1y, \varepsilon, xw_3) \curvearrowright (q', \ominus, w_1, \varepsilon, w_3)$, where $q' \in \delta(q, x, y, \ominus)$.
3. $\oplus$-jumping: $(q, \oplus, w_1, uw_2v, w_3) \curvearrowright (q, s, w_1u, w_2, vw_3)$, where s is either $\oplus$ if $|w_2| > 0$ or $\ominus$.
4. $\ominus$-jumping: $(q, \ominus, w_1\{\#\}^*, \varepsilon, \{\#\}^*w_3) \curvearrowright (q, \ominus, w_1, \varepsilon, w_3)$.

Accepted language $L(M)$

A string w is accepted by a jumping $5' \to 3'$ WK automaton M if and only if $(q_0, \oplus, \varepsilon, w, \varepsilon) \curvearrowright^* (q_f, \ominus, \varepsilon, \varepsilon, \varepsilon)$, for $q_f \in F$.

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Examples – Input 1

Example automaton $L(M) = \{w \in \{a, b\}^* : |w|_a = |w|_b\}$

$$M = (\{a, b\}, \{s\}, s, \{s\}, \delta)$$

where δ:

$$\delta(s, a, b, \oplus) = \{s\}$$
$$\delta(s, a, b, \ominus) = \{s\}$$

Input *aaabbb*

$$(s, \oplus, \varepsilon, aaabbb, \varepsilon) \curvearrowright$$
$$(s, \oplus, \#, aabb, \#) \curvearrowright$$
$$(s, \oplus, \#\#, ab, \#\#) \curvearrowright$$
$$(s, \ominus, \#\#\#, \varepsilon, \#\#\#) \curvearrowright$$
$$(s, \ominus, \varepsilon, \varepsilon, \varepsilon)$$

Examples – Input 2

Example automaton $L(M) = \{w \in \{a,b\}^* : |w|_a = |w|_b\}$

$$M = (\{a,b\},\{s\},s,\{s\},\delta)$$

where δ:

$$\delta(s,a,b,\oplus) = \{s\}$$
$$\delta(s,a,b,\ominus) = \{s\}$$

Input *baabba*

$$(s,\oplus,\varepsilon,baabba,\varepsilon) \curvearrowright$$
$$(s,\oplus,b,aabb,a) \curvearrowright$$
$$(s,\oplus,b\#,ab,\#a) \curvearrowright$$
$$(s,\ominus,b\#\#,\varepsilon,\#\#a) \curvearrowright$$
$$(s,\ominus,b,\varepsilon,a) \curvearrowright$$
$$(s,\ominus,\varepsilon,\varepsilon,\varepsilon)$$

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General results

Lemma 5.1 For every regular language L, there is a jumping $5' \to 3'$ WK automaton M such that $L = L(M)$.

Usually does not hold in JFA, but we can simulate classical FA.

Lemma 5.2 For every sensing $5' \to 3'$ WK automaton M_1, there is a jumping $5' \to 3'$ WK automaton M_2 such that $L(M_1) = L(M_2)$.

M can model linear languages with $\oplus$-reading steps.

Theorem 5.3 $\mathbf{LIN} = \mathbf{SWK} \subset \mathbf{JWK}$.

SWK – the language family of sensing $5' \to 3'$ WKA
JWK – the language family of jumping $5' \to 3'$ WKA

General results

The next two characteristics follow from the previous results.

Theorem 5.4 Jumping $5' \rightarrow 3'$ WK automata without $\ominus$-reading steps accept linear languages.

If $\ominus$-reading is not used, M can be simulated with a linear grammar.

Proposition 5.5 The language family accepted by double-jumping finite automata that perform right-left and left-right jumps is strictly included in **JWK**.

It was previously shown that these families are strictly included in **LIN**.

General results

Lemma 5.10 There are some non-context-free languages accepted by jumping $5' \to 3'$ WK automata.

$L(M) = \{w_1 w_2 : w_1 \in \{a, b\}^*,\ w_2 \in \{c, d\}^*,\ |w_1|_a = |w_2|_c,\ |w_1|_b = |w_2|_d\}$

Lemma 5.6

There is no jumping $5' \to 3'$ WK automaton M such that $L(M) = \{a^n b^n c^n : n \geq 0\}$.

Lemma 5.7

There is no jumping $5' \to 3'$ WK automaton M such that $L(M) = \{w \in \{a, b, c\}^* : |w|_a = |w|_b = |w|_c\}$.

Lemma 5.11

There is no jumping $5' \to 3'$ WK automaton M such that $L(M) = \{a^n b^n c^m d^m : n, m \geq 0\}$.

General results

Proposition 5.8 JWK is incomparable with GJFA and JFA.

$\{w \in \{a, b, c\}^* : |w|_a = |w|_b = |w|_c\}$
$\{a^n b^n : n \geq 0\}$

Theorem 5.9 JWK $\subset$ CS.

simulated by linear bounded automata

Theorem 5.12 JWK and CF are incomparable.

Lemma 5.10, Lemma 5.11

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Restrictions in Watson-Crick automata

Definition

- **N** stateless, i.e., with only one state: if $Q = F = \{q_0\}$
- **F** all-final, i.e., with only final states: if $Q = F$
- **S** simple (at most one head moves in a step)
  $\delta \colon (Q \times (\binom{V^*}{\{\varepsilon\}} \cup \binom{\{\varepsilon\}}{V^*})) \to 2^Q$
- **1** 1-limited (exactly one letter is being read in a step)
  $\delta \colon (Q \times (\binom{V}{\{\varepsilon\}} \cup \binom{\{\varepsilon\}}{V})) \to 2^Q$

Further variations such as **NS**, **FS**, **N1**, and **F1** WK automata can be identified in a straightforward way by using multiple constraints.

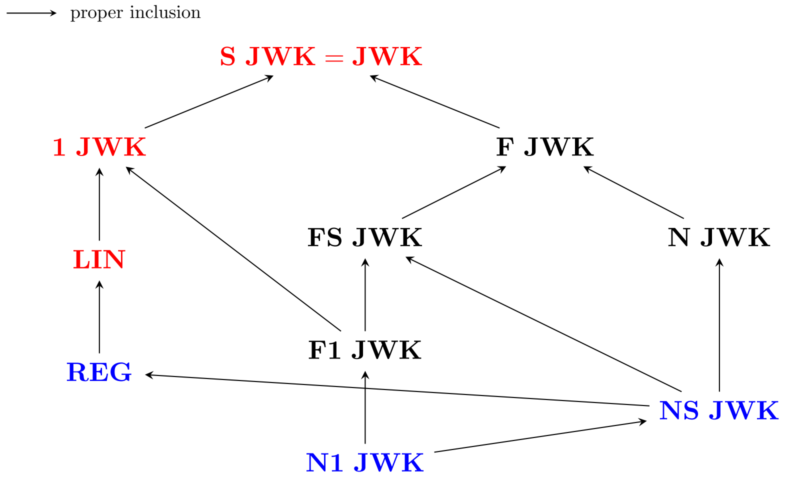
Previous results with restricted variations

Sensing $5' \rightarrow 3'$ Watson-Crick Automata (without the sensing distance)

$\longrightarrow$ proper inclusion

$$
\begin{tikzcd}
WK = LIN = S\,WK = 1\,WK & \\
F\,WK \arrow[u] & \\
& N\,WK \arrow[ul] \\
FS\,WK \arrow[uu] & \\
& NS\,WK \arrow[ul] \arrow[uu] \\
F1\,WK \arrow[uu] & \\
& N1\,WK \arrow[ul] \arrow[uu]
\end{tikzcd}
$$

Results on restricted variations



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Accepting power

- increased above sensing $5' \to 3'$ WK automata
- some non-linear and even some non-context-free languages
- the jumping movement of the heads is restricted compared to JFA: limited capabilities to accept languages that require discontinuous information processing

Open Question – Full-reading sensing $5' \to 3'$ WK automata

There are some languages accepted by full-reading sensing $5' \to 3'$ WK automata that cannot be accepted by jumping $5' \to 3'$ WK automata. But what about the other direction?

Alternative definition

Open Question – Can we somehow safely remove $\#$ and $\ominus$-jumping steps from the model?

The answer is yes, if the model uses 1-limited restriction.
But what about the general case?

Thank you!
Any questions?