CHARACTERIZATIONS OF LRR- LANGUAGES BY CORRECTNESS-PRESERVING COMPUTATIONS

František Mráz, Friedrich Otto, and Martin Plátek

Charles University, Prague, Czech Republic

NCMA 2018

Košice

August 21–22, 2018

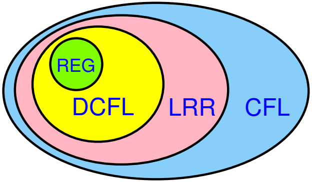
The main result

A superclass of DCFL can be accepted by “well-behaved” restarting automata

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

the class of left-to-right regular languages (LRR)



- DCFL can be accepted deterministically by LR(1)-analyzers with lookahead of size 1
- LRR can be accepted deterministically by LR-analyzers with unlimited lookahead; example

$$\{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$$

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving
- even more restrictions are possible

The main result

A superclass of DCFL can be accepted by “well-behaved” restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving
- even more restrictions are possible
    - deleting only – moreover, deleting at most two continuous factor in one step

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving
- even more restrictions are possible
  - deleting only – moreover, deleting at most two continuous factor in one step
  - always reduce the input into a word of small size (limited by a constant) before accepting or rejecting

The main result

A superclass of DCFL can be accepted by “well-behaved” restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving
- even more restrictions are possible
  - deleting only – moreover, deleting at most two continuous factor in one step
  - always reduce the input into a word of small size (limited by a constant) before accepting or rejecting
    - expresses the structure for correct inputs

The main result

A superclass of DCFL can be accepted by "well-behaved" restarting automata

- deterministic
- no auxiliary symbols
- length-reducing
- local changes
- monotone
- correctness-preserving
- even more restrictions are possible
  - deleting only – moreover, deleting at most two continuous factor in one step
  - always reduce the input into a word of small size (limited by a constant) before accepting or rejecting
    - expresses the structure for correct inputs
    - shows a core for an error in rejected inputs

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,

Two-Way Restarting List Automaton

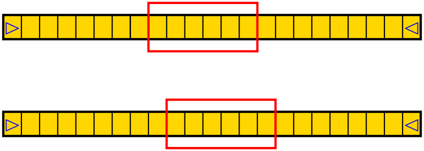
$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$

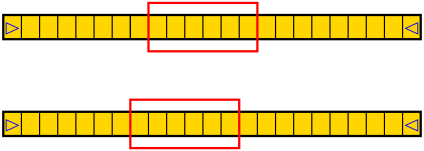
- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\rhd$ and $\lhd$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.



Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

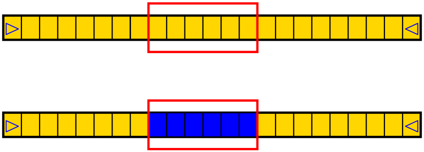
- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.



Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

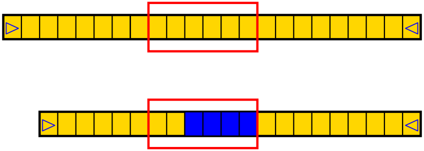
- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.



Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$

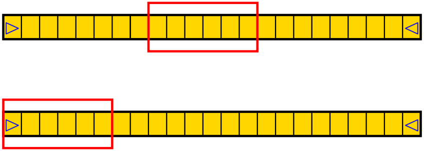
- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\rhd$ and $\lhd$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.



Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.



Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\triangleright$ and $\triangleleft$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.

Two-Way Restarting List Automaton

$M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$

- finite state control, set of states Q,
- read/write window of a fixed size k,
- tape = list of symbols delimited by sentinels $\rhd$ and $\lhd$,
- input alphabet Σ,
- working alphabet Γ, $\Sigma \subseteq \Gamma$,
- operations: MVR, MVL, W (v), SL (v), Restart, Accept, Reject.

Accepted Languages

- a *configuration* of an RLA M: $\alpha q \beta$
  - q is the current state
  - $\alpha\beta \in \{\triangleright\} \cdot \Gamma^* \cdot \{\triangleleft\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β

Accepted Languages

- a *configuration* of an RLA M: $\alpha q \beta$
  - q is the current state
  - $\alpha\beta \in \{\rhd\} \cdot \Gamma^* \cdot \{\lhd\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β
- a *restarting configuration*: $q_0 \rhd w \lhd$, where $w \in \Gamma^*$

Accepted Languages

- a *configuration* of an RLA M: $\alpha q \beta$
  - q is the current state
  - $\alpha\beta \in \{\triangleright\} \cdot \Gamma^* \cdot \{\triangleleft\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β
- a *restarting configuration*: $q_0 \triangleright w \triangleleft$, where $w \in \Gamma^*$
- an *initial configuration*: $q_0 \triangleright w \triangleleft$, where $w \in \Sigma^*$

Accepted Languages

- a *configuration* of an RLA M: $\alpha q \beta$
  - q is the current state
  - $\alpha\beta \in \{\rhd\} \cdot \Gamma^* \cdot \{\lhd\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β
- a *restarting configuration*: $q_0 \rhd w \lhd$, where $w \in \Gamma^*$
- an *initial configuration*: $q_0 \rhd w \lhd$, where $w \in \Sigma^*$
- the input language of M:

$$L(M) = \{w \in \Sigma^* \mid q_0 \rhd w \lhd \vdash_M^* \text{Accept}\}.$$

Accepted Languages

- a *configuration* of an RLA M: $\alpha q\beta$
  - q is the current state
  - $\alpha\beta \in \{\rhd\} \cdot \Gamma^* \cdot \{\lhd\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β
- a *restarting configuration*: $q_0 \rhd w \lhd$, where $w \in \Gamma^*$
- an *initial configuration*: $q_0 \rhd w \lhd$, where $w \in \Sigma^*$
- the input language of M:

$$L(M) = \{w \in \Sigma^* \mid q_0 \rhd w \lhd \vdash_M^* \text{Accept}\}.$$

- the basic (characteristic) language of M:

$$L_{\mathrm{C}}(M) = \{w \in \Gamma^* \mid q_0 \rhd w \lhd \vdash_M^* \text{Accept}\}.$$

Accepted Languages

- a *configuration* of an RLA M: $\alpha q \beta$
  - q is the current state
  - $\alpha\beta \in \{\triangleright\} \cdot \Gamma^* \cdot \{\triangleleft\}$ is the current contents of the tape
  - contents of the window = the first k symbols of β
- a *restarting configuration*: $q_0 \triangleright w \triangleleft$, where $w \in \Gamma^*$
- an *initial configuration*: $q_0 \triangleright w \triangleleft$, where $w \in \Sigma^*$
- the input language of M:

$$L(M) = \{w \in \Sigma^* \mid q_0 \triangleright w \triangleleft \vdash_M^* \mathrm{Accept}\}.$$

- the basic (characteristic) language of M:

$$L_{\mathrm{C}}(M) = \{w \in \Gamma^* \mid q_0 \triangleright w \triangleleft \vdash_M^* \mathrm{Accept}\}.$$

- obviously $L_{\mathrm{C}}(M) \cap \Sigma^* = L(M)$.

Cycles, Reductions

Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step

Cycles, Reductions

Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step
- a tail = a part of a computation after the last restarting configuration and a halting step (Accept or Reject)

Cycles, Reductions
Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step
- a tail = a part of a computation after the last restarting configuration and a halting step (Accept or Reject)
- notation:
  - $q_0 \rhd u \lhd \vdash^{c}_{M} q_0 \rhd v \lhd$ denotes a cycle of M
  - then we write $u \Rightarrow^{c}_{M} v$ – the cycle rewriting relation of M

Cycles, Reductions
Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step
- a tail = a part of a computation after the last restarting configuration and a halting step (Accept or Reject)
- notation:
  - $q_0 \rhd u \lhd \vdash_M^c q_0 \rhd v \lhd$ denotes a cycle of M
  - then we write $u \Rightarrow_M^c v$ – the cycle rewriting relation of M
- if $u \Rightarrow_M^c v$ and $|u| > |v|$, then $u \Rightarrow_M^c v$ is called a *reduction*

Cycles, Reductions
Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step
- a tail = a part of a computation after the last restarting configuration and a halting step (Accept or Reject)
- notation:
  - $q_0 \rhd u \lhd \vdash_M^c q_0 \rhd v \lhd$ denotes a cycle of M
  - then we write $u \Rightarrow_M^c v$ – the cycle rewriting relation of M
- if $u \Rightarrow_M^c v$ and $|u| > |v|$, then $u \Rightarrow_M^c v$ is called a *reduction*

Fact 1

(Error Preserving Property for basic languages of RLAs).
Let M be an RLA. If $u \Rightarrow_M^{c^*} v$ and $u \notin L_{\mathrm{C}}(M)$, then $v \notin L_{\mathrm{C}}(M)$.

Cycles, Reductions
Error and Correctness Preserving Properties

- a cycle = a part of a computation between a restarting configuration and the configuration after a restart step
- a tail = a part of a computation after the last restarting configuration and a halting step (Accept or Reject)
- notation:
  - $q_0 \rhd u \lhd \vdash^c_M q_0 \rhd v \lhd$ denotes a cycle of M
  - then we write $u \Rightarrow^c_M v$ – the cycle rewriting relation of M
- if $u \Rightarrow^c_M v$ and $|u| > |v|$, then $u \Rightarrow^c_M v$ is called a *reduction*

Fact 1

(Error Preserving Property for basic languages of RLAs).
Let M be an RLA. If $u \Rightarrow^{c*}_M v$ and $u \notin L_{\mathrm{C}}(M)$, then $v \notin L_{\mathrm{C}}(M)$.

Fact 2

(Correctness Preserving Property for basic languages of det-RLAs).
Let M be a deterministic RLA. If $u \Rightarrow^{c*}_M v$ and $u \in L_{\mathrm{C}}(M)$, then $v \in L_{\mathrm{C}}(M)$.

Refinements and Constraints on RLAs

- Restricted SL-steps:
  - A delete-left step (DL-step) = an SL-step which can only delete symbols
  - A contextual-left step (CL-step) = an DL-step which can delete at most two factors
- Notation for $T \subseteq \{\text{MVR, MVL, W, SL, DL, CL, Restart}\}$: T-RLA denotes RLAs which can use operations from $T \cup \{\text{Accept}, \text{Reject}\}$

Refinements and Constraints on RLAs

Monotonicity and Complete Monotonicity

- Let $C = C_k, C_{k+1}, \dots, C_j$ be a subcomputation and let $C_w = \triangleright\alpha q\beta\triangleleft$ be a configuration from C. Then $D_r(C_w) = |\beta \triangleleft|$ is the *right distance* of C_w.

Refinements and Constraints on RLAs

Monotonicity and Complete Monotonicity

- Let $C = C_k, C_{k+1}, \ldots, C_j$ be a subcomputation and let $C_w = \triangleright\alpha q\beta\triangleleft$ be a configuration from C. Then $D_r(C_w) = |\beta \triangleleft|$ is the *right distance* of C_w.
- Monotonicity of rewritings: let $C_1, \ldots, C_n$ be a maximal subsequence of C containing all configurations in which a rewriting occurs.
  C is monotone if $D_r(C_1) \geq D_r(C_2) \geq \cdots \geq D_r(C_n)$.
  M is monotone if all its computations are monotone.

Refinements and Constraints on RLAs

Monotonicity and Complete Monotonicity

- Let $C = C_k, C_{k+1}, \ldots, C_j$ be a subcomputation and let $C_w = \rhd\alpha q\beta\lhd$ be a configuration from C. Then $D_r(C_w) = |\beta \lhd|$ is the *right distance* of C_w.
- Monotonicity of rewritings: let $C_1, \ldots, C_n$ be a maximal subsequence of C containing all configurations in which a rewriting occurs.
  C is monotone if $D_r(C_1) \geq D_r(C_2) \geq \cdots \geq D_r(C_n)$.
  M is monotone if all its computations are monotone.
- M is completely monotone if $D_r(C_\ell) \geq D_r(C_{\ell+1})$ holds whenever configuration $C_\ell \vdash_M C_{\ell+1}$.

Refinements and Constraints on RLAs

Monotonicity and Complete Monotonicity

- Let $C = C_k, C_{k+1}, \ldots, C_j$ be a subcomputation and let $C_w = \triangleright\alpha q\beta\triangleleft$ be a configuration from C. Then $D_r(C_w) = |\beta \triangleleft|$ is the *right distance* of C_w.
- Monotonicity of rewritings: let $C_1, \ldots, C_n$ be a maximal subsequence of C containing all configurations in which a rewriting occurs.
  C is monotone if $D_r(C_1) \geq D_r(C_2) \geq \cdots \geq D_r(C_n)$.
  M is monotone if all its computations are monotone.
- M is completely monotone if $D_r(C_\ell) \geq D_r(C_{\ell+1})$ holds whenever configuration $C_\ell \vdash_M C_{\ell+1}$.

Fact 3

Each $\{$MVR,SL,W$\}$*-automaton is completely monotone.*

Refinements and Constraints on RLAs
Monotonicity and Complete Monotonicity

- Let $C = C_k, C_{k+1}, \ldots, C_j$ be a subcomputation and let $C_w = \triangleright\alpha q\beta\triangleleft$ be a configuration from C. Then $D_r(C_w) = |\beta\triangleleft|$ is the *right distance* of C_w.
- Monotonicity of rewritings: let $C_1, \ldots, C_n$ be a maximal subsequence of C containing all configurations in which a rewriting occurs.
  C is monotone if $D_r(C_1) \geq D_r(C_2) \geq \cdots \geq D_r(C_n)$.
  M is monotone if all its computations are monotone.
- M is completely monotone if $D_r(C_\ell) \geq D_r(C_{\ell+1})$ holds whenever configuration $C_\ell \vdash_M C_{\ell+1}$.

Fact 3

Each {MVR,SL,W}*-automaton is completely monotone.*

- A {MVR,SL,W}-automaton with a window of size $k \geq 2$ can be interpreted as a pushdown automaton with a k-lookahead and with a limited look under the top of the pushdown.
- A deterministic PDA can be simulated by a det-{MVR,SL,W}-automaton with a window of size 1.

RLWW-automata

- An RLWW-*automaton* M is an RLA
  1. No W-steps (rewritings only by SL-steps).
  2. Exactly one SL-step in each cycle.
  3. At most one SL-step in each tail computation.

RLWW-automata

- An RLWW-*automaton* M is an RLA
  1. No W-steps (rewritings only by SL-steps).
  2. Exactly one SL-step in each cycle.
  3. At most one SL-step in each tail computation.
- For RLWW-automata, all cycle-rewritings are reductions.

RLWW-automata

- An RLWW-*automaton* M is an RLA
  1. No W-steps (rewritings only by SL-steps).
  2. Exactly one SL-step in each cycle.
  3. At most one SL-step in each tail computation.
- For RLWW-automata, all cycle-rewritings are reductions.
- Different variants of RLWW-automata

		SL-steps	DL-steps only	CL-steps only
auxiliary symbols possible ($-$WW)	MVL-steps (RL$-$)	RLWW	RLWWD	RLWWC
	no MVL-steps, rewrite followed by restart (R$-$)	RWW	RWWD	RWWC
no auxiliary symbols ($-$W)	MVL-steps (RL$-$)	RLW	RLWD	RLWC
	no MVL-steps, rewrite followed by restart (R$-$)	RW	RWD	RWC

- For each RLW-automaton M, $L(M) = L_C(M)$.

Correctness Preserving Properties

(a) An RLA-automaton M satisfies the *Complete Weak Correctness Preserving Property (CWCPP) for its basic (input) language* if, for each accepting computation $C_0, C_1, \ldots, C_n$ of M, $u_j \in L_{\mathrm{C}}(M)$ ($u_j \in L(M)$) for all $j = 0, 1, \ldots, n$, where u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

Correctness Preserving Properties

(a) An RLA-automaton M satisfies the *Complete Weak Correctness Preserving Property (CWCPP) for its basic (input) language* if, for each accepting computation $C_0, C_1, \ldots, C_n$ of M, $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, where u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

(b) An RLA-automaton M satisfies the *Complete Strong Correctness Preserving Property (CSCPP) for its basic (input) language* if, for each computation $C_0, C_1, \ldots, C_n$ of M, we have that $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, if $u_i \in L_{\mathrm{C}}(M)$ $(u_i \in L(M))$ for some i. Here u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

Correctness Preserving Properties

(a) An RLA-automaton M satisfies the *Complete Weak Correctness Preserving Property (CWCPP) for its basic (input) language* if, for each accepting computation $C_0, C_1, \ldots, C_n$ of M, $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, where u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

(b) An RLA-automaton M satisfies the *Complete Strong Correctness Preserving Property (CSCPP) for its basic (input) language* if, for each computation $C_0, C_1, \ldots, C_n$ of M, we have that $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, if $u_i \in L_{\mathrm{C}}(M)$ $(u_i \in L(M))$ for some i. Here u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

- Complete $\equiv$ each and every operation of the automaton M considered preserves the property of the tape contents to belong to the language $L_{\mathrm{C}}(M)$ $(L(M))$.

Correctness Preserving Properties

(a) An RLA-automaton M satisfies the *Complete Weak Correctness Preserving Property (CWCPP) for its basic (input) language* if, for each accepting computation $C_0, C_1, \ldots, C_n$ of M, $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, where u_j is the contents of the tape in configuration C_j $(0 \le j \le n)$.

(b) An RLA-automaton M satisfies the *Complete Strong Correctness Preserving Property (CSCPP) for its basic (input) language* if, for each computation $C_0, C_1, \ldots, C_n$ of M, we have that $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, if $u_i \in L_{\mathrm{C}}(M)$ $(u_i \in L(M))$ for some i. Here u_j is the contents of the tape in configuration C_j $(0 \le j \le n)$.

- Complete $\equiv$ each and every operation of the automaton M considered preserves the property of the tape contents to belong to the language $L_{\mathrm{C}}(M)$ $(L(M))$.
- No intermediate information is stored on the tape.

Correctness Preserving Properties

(a) An RLA-automaton M satisfies the *Complete Weak Correctness Preserving Property (CWCPP) for its basic (input) language* if, for each accepting computation $C_0, C_1, \ldots, C_n$ of M, $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, where u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

(b) An RLA-automaton M satisfies the *Complete Strong Correctness Preserving Property (CSCPP) for its basic (input) language* if, for each computation $C_0, C_1, \ldots, C_n$ of M, we have that $u_j \in L_{\mathrm{C}}(M)$ $(u_j \in L(M))$ for all $j = 0, 1, \ldots, n$, if $u_i \in L_{\mathrm{C}}(M)$ $(u_i \in L(M))$ for some i. Here u_j is the contents of the tape in configuration C_j $(0 \leq j \leq n)$.

- Complete $\equiv$ each and every operation of the automaton M considered preserves the property of the tape contents to belong to the language $L_{\mathrm{C}}(M)$ $(L(M))$.
- No intermediate information is stored on the tape.
- Complete Weak and Strong Correctness Preserving Properties do not depend on the operation of restart.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a monotone RWW-automaton M; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a monotone RWW-automaton M; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- either
  - performs the cycle $q_0 \rhd a^m b^n x \lhd \vdash_M^c q_0 \rhd a^{m-1} C b^{n-1} x \lhd$, where C is an auxiliary symbol (guessing that $x = c$);
  - repeatedly performs CL-steps rewriting aCb into C
  - accepts on $\rhd Cc \lhd$

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a monotone RWW-automaton M; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- either
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash_M^c q_0 \triangleright a^{m-1} C b^{n-1} x \triangleleft$, where C is an auxiliary symbol (guessing that $x = c$);
  - repeatedly performs CL-steps rewriting aCb into C
  - accepts on $\triangleright Cc \triangleleft$
- or
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash_M^c q_0 \triangleright a^{m-1} D b^{n-2} x \triangleleft$, where D is an auxiliary symbol (guessing that $x = d$);
  - repeatedly performs CL-steps rewriting $aDbb$ into D
  - accepts on $\triangleright Dd \triangleleft$

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a monotone RWW-automaton M; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- either
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash^c_M q_0 \triangleright a^{m-1} C b^{n-1} x \triangleleft$, where C is an auxiliary symbol (guessing that $x = c$);
  - repeatedly performs CL-steps rewriting aCb into C
  - accepts on $\triangleright Cc \triangleleft$
- or
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash^c_M q_0 \triangleright a^{m-1} D b^{n-2} x \triangleleft$, where D is an auxiliary symbol (guessing that $x = d$);
  - repeatedly performs CL-steps rewriting $aDbb$ into D
  - accepts on $\triangleright Dd \triangleleft$
- In an accepting computation of M, all but the initial configuration contain an occurrence of an auxiliary symbol $\Rightarrow$ does not satisfy the Complete Weak Correctness Preserving Property for its input language.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a monotone RWW-automaton M; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- either
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash_M^c q_0 \triangleright a^{m-1} C b^{n-1} x \triangleleft$, where C is an auxiliary symbol (guessing that $x = c$);
  - repeatedly performs CL-steps rewriting aCb into C
  - accepts on $\triangleright Cc \triangleleft$
- or
  - performs the cycle $q_0 \triangleright a^m b^n x \triangleleft \vdash_M^c q_0 \triangleright a^{m-1} D b^{n-2} x \triangleleft$, where D is an auxiliary symbol (guessing that $x = d$);
  - repeatedly performs CL-steps rewriting $aDbb$ into D
  - accepts on $\triangleright Dd \triangleleft$
- In an accepting computation of M, all but the initial configuration contain an occurrence of an auxiliary symbol $\Rightarrow$ does not satisfy the Complete Weak Correctness Preserving Property for its input language.
- The automaton is monotone.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a deterministic monotone RLW-automaton M'; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- Scans the given input completely and accepts or rejects words of length 1.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a deterministic monotone RLW-automaton M'; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- Scans the given input completely and accepts or rejects words of length 1.
- If the last letter is a c, it deletes ab and restarts;
  if the last letter is a d, it deletes abb and restarts.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a deterministic monotone RLW-automaton M'; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- Scans the given input completely and accepts or rejects words of length 1.
- If the last letter is a c, it deletes ab and restarts;
  if the last letter is a d, it deletes abb and restarts.
- The automaton is monotone.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a deterministic monotone RLW-automaton M'; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- Scans the given input completely and accepts or rejects words of length 1.
- If the last letter is a c, it deletes ab and restarts;
  if the last letter is a d, it deletes abb and restarts.
- The automaton is monotone.
- The (accepting) computations of M' are much more transparent than those of the RWW-automaton M.

Example

$L = \{a^n b^n c, a^n b^{2n} d \mid n \geq 0\}$
is accepted by a deterministic monotone RLW-automaton M'; on input $a^m b^n x$, where $m, n \geq 2$ and $x \in \{c, d\}$:

- Scans the given input completely and accepts or rejects words of length 1.
- If the last letter is a c, it deletes ab and restarts;
  if the last letter is a d, it deletes abb and restarts.
- The automaton is monotone.
- The (accepting) computations of M' are much more transparent than those of the RWW-automaton M.
- The det-RLW-automaton M' satisfies the Complete Strong Correctness Preserving Property for its input language.

Complete Correctness Preserving Properties for RLWW-Automata

- Each RLW-automaton can be turned into an RLW-automaton that satisfies the Complete Weak Correctness Preserving Property for its input language.
  - Take care of tails – do not rewrite in tails.
- Each deterministic RLW-automaton can be turned into a deterministic RLW-automaton that satisfies the Complete Strong Correctness Preserving Property for its input language.

Complete Correctness Preserving Properties for RLWW-Automata

- Each RLW-automaton can be turned into an RLW-automaton that satisfies the Complete Weak Correctness Preserving Property for its input language.
  - Take care of tails – do not rewrite in tails.
- Each deterministic RLW-automaton can be turned into a deterministic RLW-automaton that satisfies the Complete Strong Correctness Preserving Property for its input language.

Characterization of LRR

- Already known:
  (a) [Jančar, Mráz, Plátek, Vogel, '99]
  DCFL $= \mathcal{L}(\text{det-mon-RWC}) \subsetneq \mathcal{L}(\text{det-mon-RLWC})$

- New:

Characterization of LRR

- Already known:
  (a) [Jančar, Mráz, Plátek, Vogel, '99]
  DCFL $= \mathcal{L}$(det-mon-RWC) $\subsetneq \mathcal{L}$(det-mon-RLWC)
  (b) [Otto,'09] LRR $= \mathcal{L}$(det-mon-RLWW) $= \mathcal{L}$(det-mon-RLWD)
- New:

Characterization of LRR

- Already known:
  (a) [Jančar, Mráz, Plátek, Vogel, '99]
  DCFL $= \mathcal{L}$(det-mon-RWC) $\subsetneq \mathcal{L}$(det-mon-RLWC)
  (b) [Otto,'09] LRR $= \mathcal{L}$(det-mon-RLWW) $= \mathcal{L}$(det-mon-RLWD)
- New:

Theorem 4

For each det-mon-RLWW-automaton M_a, there exists a det-mon-RLWC-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

Characterization of LRR

- Already known:
  - (a) [Jančar, Mráz, Plátek, Vogel, '99]
    DCFL $= \mathcal{L}(\text{det-mon-RWC}) \subsetneq \mathcal{L}(\text{det-mon-RLWC})$
  - (b) [Otto,'09] LRR $= \mathcal{L}(\text{det-mon-RLWW}) = \mathcal{L}(\text{det-mon-RLWD})$
- New:

Theorem 4

For each det-mon-RLWW-automaton M_a, there exists a det-mon-RLWC-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

- $L = L(M_a)$ belongs to the class LRR

Characterization of LRR

- Already known:
  (a) [Jančar, Mráz, Plátek, Vogel, '99]
  $\mathsf{DCFL} = \mathcal{L}(\text{det-mon-RWC}) \subsetneq \mathcal{L}(\text{det-mon-RLWC})$
  (b) [Otto,'09] $\mathsf{LRR} = \mathcal{L}(\text{det-mon-RLWW}) = \mathcal{L}(\text{det-mon-RLWD})$
- New:

Theorem 4

For each det-mon-RLWW-automaton M_a, there exists a det-mon-RLWC-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

- $L = L(M_a)$ belongs to the class LRR
- [Čulík II, Cohen, '73] there exists a deterministic sequential right-to-left transducer G such that $L_1 = G(L)$ is a deterministic context-free language.

Characterization of LRR

- Already known:
  - (a) [Jančar, Mráz, Plátek, Vogel, '99] $\text{DCFL} = \mathcal{L}(\text{det-mon-RWC}) \subsetneq \mathcal{L}(\text{det-mon-RLWC})$
  - (b) [Otto,'09] $\text{LRR} = \mathcal{L}(\text{det-mon-RLWW}) = \mathcal{L}(\text{det-mon-RLWD})$
- New:

Theorem 4

For each det-mon-RLWW-automaton M_a, there exists a det-mon-RLWC-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

- $L = L(M_a)$ belongs to the class LRR
- [Čulík II, Cohen, '73] there exists a deterministic sequential right-to-left transducer G such that $L_1 = G(L)$ is a deterministic context-free language.
- We construct a $\{\text{MVR}, \text{MVL}, \text{W}, \text{CL}, \text{Restart}\}$-automaton M_2 such that M_2 accepts on input w iff $G(w) \in L_1$ iff $w \in L$.

Characterization of LRR

- Already known:
  - (a) [Jančar, Mráz, Plátek, Vogel, '99] DCFL $= \mathcal{L}$(det-mon-RWC) $\subsetneq \mathcal{L}$(det-mon-RLWC)
  - (b) [Otto,'09] LRR $= \mathcal{L}$(det-mon-RLWW) $= \mathcal{L}$(det-mon-RLWD)
- New:

Theorem 4

For each det-mon-RLWW-automaton M_a, there exists a det-mon-RLWC-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

- $L = L(M_a)$ belongs to the class LRR
- [Čulík II, Cohen, '73] there exists a deterministic sequential right-to-left transducer G such that $L_1 = G(L)$ is a deterministic context-free language.
- We construct a $\{$MVR, MVL, W, CL, Restart$\}$-automaton M_2 such that M_2 accepts on input w iff $G(w) \in L_1$ iff $w \in L$.
- We simulate M_2 by a det-mon-RLWC-automaton M_3.

Characterization of LRR

- $L = L(M_a)$ belongs to the class LRR
- [Čulík II, Cohen, '73] there exists a deterministic sequential right-to-left transducer G such that $L_1 = G(L)$ is a deterministic context-free language.

$w =$	a_1	$\dots$	a_{n-1}	a_n	
	a_1	$\dots$	a_{n-1}	a_n, p_n	
	a_1	$\dots$	a_{n-1}, p_{n-1}	a_n, p_n	
	a_1, p_1	$\dots$	a_{n-1}, p_{n-1}	a_n, p_n	$= G(w)$

- it can be simulated by a $\{\mathrm{MVR}, \mathrm{MVL}, W\}$-RLA
- $L_1 = G(L)$ is from DCFL $\Rightarrow$ it is accepted by a det-mon-RWC-automaton M_1 – operations $\{\mathrm{MVR}, \mathrm{CL}, \mathrm{Restart}\}$
- an $\{\mathrm{MVR}, \mathrm{MVL}, \mathrm{W}, \mathrm{CL}, \mathrm{Restart}\}$-automaton M_2 can simulate the composition of G and M_1

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1
  - The first step of M_1 on $G(w)$ can be simulated,

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1
  - The first step of M_1 on $G(w)$ can be simulated,
  - but if M_1 perform a MVR-step, it must reconstruct the contents of the window

Characterization of LRR

- the $\{\mathrm{MVR}, \mathrm{MVL}, \mathrm{W}, \mathrm{CL}, \mathrm{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1
  - The first step of M_1 on $G(w)$ can be simulated,
  - but if M_1 perform a MVR-step, it must reconstruct the contents of the window
  - for that a mirrored version of a theorem [Aho, Hopcroft, Ullman,'69] can be used

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1
  - The first step of M_1 on $G(w)$ can be simulated,
  - but if M_1 perform a MVR-step, it must reconstruct the contents of the window
  - for that a mirrored version of a theorem [Aho, Hopcroft, Ullman,'69] can be used
    - for each (one-way) deterministic finite-state automaton A there exists a two-way deterministic finite-state automaton B such that if A arrives at some position i of its input x in a state q_i, then B starting at the same position in state q_i, finishes at position $i-1$ in the state q_{i-1}

Characterization of LRR

- the $\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{W}, \mathsf{CL}, \mathsf{Restart}\}$-automaton M_2 can be simulated by a det-mon-RLWC-automaton M_3.
  - M_3 behaves like M_2, but without rewrites of the form $a \to (a, p_a)$
  - It scans tape from right to left and simulates the transducer G in its finite-state control and remembers the output of G just for the letters inside the window of M_1
  - The first step of M_1 on $G(w)$ can be simulated,
  - but if M_1 perform a MVR-step, it must reconstruct the contents of the window
  - for that a mirrored version of a theorem [Aho, Hopcroft, Ullman,'69] can be used
    - for each (one-way) deterministic finite-state automaton A there exists a two-way deterministic finite-state automaton B such that if A arrives at some position i of its input x in a state q_i, then B starting at the same position in state q_i, finishes at position $i-1$ in the state q_{i-1}
  - The first cycle of M_1 on $G(w)$ can be simulated; the resulting contents of the tape x is such that $G(x) \in L_1$ iff $G(w) \in L_1$ hence $x \in L$ iff $w \in L$.

Corollary 5

Each det-mon-RLWC-automaton can be turned into det-mon-$\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{CL}\}$-automaton satisfying the Complete Strong Correctness Preserving Property for its input language.

Proof:

- Each deterministic RLWC-automaton satisfies the Complete Strong Correctness Preserving Property for its input language.
- An RLWC-automaton can use MVR-, CL- and Restart-steps only
- Simulate each Restart-step by MVL-steps!

Corollary 5

Each det-mon-RLWC-automaton can be turned into det-mon-$\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{CL}\}$-automaton satisfying the Complete Strong Correctness Preserving Property for its input language.

Proof:

- Each deterministic RLWC-automaton satisfies the Complete Strong Correctness Preserving Property for its input language.
- An RLWC-automaton can use MVR-, CL- and Restart-steps only
- Simulate each Restart-step by MVL-steps!

Proposition 1

For each det-mon-$\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{SL}\}$-automaton M_a, there exists a det-mon-RLWW-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

Corollary 5

Each det-mon-RLWC-automaton can be turned into det-mon-$\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{CL}\}$-automaton satisfying the Complete Strong Correctness Preserving Property for its input language.

Proof:

- Each deterministic RLWC-automaton satisfies the Complete Strong Correctness Preserving Property for its input language.
- An RLWC-automaton can use MVR-, CL- and Restart-steps only
- Simulate each Restart-step by MVL-steps!

Proposition 1

For each det-mon-$\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{SL}\}$-automaton M_a, there exists a det-mon-RLWW-automaton M_b such that $L(M_a) = L(M_b)$.

Proof:

- M_b must restart after simulating an SL-step of M_a
- Use the monotonicity! It encodes the state of M_a after an SL-step on its tape – together with thr rightmost symbol of the rewritten part
- The next cycle of M_b starts by finding the rightmost tape field encoding also a state

Characterisation of LRR by Automata With Complete Strong Correctness Preserving Property

cscpp- denotes the Complete Strong Correctness Preserving Property.

Corollary 6

$$\begin{aligned} \mathrm{LRR} &= \mathcal{L}(\text{det-mon-RLWW}) \\ &= \mathcal{L}(\text{det-mon-}\{\text{MVR,MVL,SL}\}) \\ &= \mathcal{L}(\text{det-mon-cscpp-RLWC}) \\ &= \mathcal{L}(\text{det-mon-cscpp-}\{\text{MVR,MVL,SL}\}) \\ &= \mathcal{L}(\text{det-mon-cscpp-}\{\text{MVR,MVL,CL}\}). \end{aligned}$$

Strong Cyclic Form

- an RLA is in *weak cyclic form* if before accepting it always shortens its tape contents so that it fits in its window

Strong Cyclic Form

- an RLA is in *weak cyclic form* if before accepting it always shortens its tape contents so that it fits in its window
- an RLA is in *strong cyclic form* if before accepting or rejecting it always shortens its tape contents so that it fits in its window

Strong Cyclic Form

- an RLA is in *weak cyclic form* if before accepting it always shortens its tape contents so that it fits in its window
- an RLA is in *strong cyclic form* if before accepting or rejecting it always shortens its tape contents so that it fits in its window

Theorem 7

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\mathrm{scf}}}^c v$.

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\mathrm{scf}}}^c v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\mathrm{scf}}}^c v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\text{scf}})$ and, for all $u \Rightarrow^{c}_{M} v$, also $u \Rightarrow^{c}_{M_{\text{scf}}} v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.
- There exists a constant c such that for each word z from $L(A_+) \cup L(A_-)$ of length at least c, there is a factorization $z = uvw$ such that $|vw| \leq c$, $|v| \geq 1$, if $z \in L(A_+)$, then $uw \in L(A_+)$ and if $z \in L(A_-)$, then $uw \in L(A_-)$

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\mathrm{scf}}}^c v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.
- There exists a constant c such that for each word z from $L(A_+) \cup L(A_-)$ of length at least c, there is a factorization $z = uvw$ such that $|vw| \leq c$, $|v| \geq 1$, if $z \in L(A_+)$, then $uw \in L(A_+)$ and if $z \in L(A_-)$, then $uw \in L(A_-)$
  1. M_{scf} accepts or rejects all “short” words

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow^{c}_{M} v$, also $u \Rightarrow^{c}_{M_{\mathrm{scf}}} v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.
- There exists a constant c such that for each word z from $L(A_+) \cup L(A_-)$ of length at least c, there is a factorization $z = uvw$ such that $|vw| \leq c$, $|v| \geq 1$, if $z \in L(A_+)$, then $uw \in L(A_+)$ and if $z \in L(A_-)$, then $uw \in L(A_-)$
  1. M_{scf} accepts or rejects all “short” words
  2. On “long” words if tests whether A_+ or A_- would accted it; if yes, it cuts out v from the tape suffix and restarts.

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\text{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\text{scf}}}^c v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.
- There exists a constant c such that for each word z from $L(A_+) \cup L(A_-)$ of length at least c, there is a factorization $z = uvw$ such that $|vw| \leq c$, $|v| \geq 1$, if $z \in L(A_+)$, then $uw \in L(A_+)$ and if $z \in L(A_-)$, then $uw \in L(A_-)$
  1. M_{scf} accepts or rejects all "short" words
  2. On "long" words if tests whether A_+ or A_- would accted it; if yes, it cuts out v from the tape suffix and restarts.
- Monotonicity is preserved.

Strong Cyclic Form

For each det-mon-RLWC-automaton M, there is a det-mon-RLWC-automaton M_{scf} in strong cyclic form such that $L(M) = L(M_{\mathrm{scf}})$ and, for all $u \Rightarrow_M^c v$, also $u \Rightarrow_{M_{\mathrm{scf}}}^c v$.

- The set of words accepted by M in a tail computation is regular – it can be accepted by a finite state automaton A_+.
- The set of words rejected by M in a tail computation is regular – it can accepted by a finite state automaton A_-.
- There exists a constant c such that for each word z from $L(A_+) \cup L(A_-)$ of length at least c, there is a factorization $z = uvw$ such that $|vw| \leq c$, $|v| \geq 1$, if $z \in L(A_+)$, then $uw \in L(A_+)$ and if $z \in L(A_-)$, then $uw \in L(A_-)$
  1. M_{scf} accepts or rejects all “short” words
  2. On “long” words if tests whether A_+ or A_- would accted it; if yes, it cuts out v from the tape suffix and restarts.
  3. Otherwise, it simulates the next cycle of M.
- Monotonicity is preserved.

RLWW-automata

Corollary 8

For all $\mathsf{Y} \in \{\lambda, \mathsf{scf}, \mathsf{scf\text{-}cscpp}\}$, *the following holds:*

$$\begin{array}{ccccc} \mathcal{L}(\text{det-mon-RLWW}) & = & \mathcal{L}(\text{det-mon-Y-RLW}) & = & \\ \mathcal{L}(\text{det-mon-Y-RLWD}) & = & \mathcal{L}(\text{det-mon-Y-RLWC}) & = & \text{LRR}. \end{array}$$

- RLWC-, RLWD-, and RLW-automata can always be modified to satisfy the Complete Weak Correctness Preserving Property for input and basic languages.
- Deterministic RLWC-, RLWD-, and RLW-automata can always be modified to satisfy the Complete Strong Correctness Preserving Property for input and basic languages.
- General – nondeterministic – RLWW-automata can be modified to satisfy CSCPP only for basic languages.

RLA-automata

Corollary 9

For all $\mathsf{X} \in \{\{\mathsf{MVR}, \mathsf{MVL}, \mathsf{SL}\}, \{\mathsf{MVR}, \mathsf{MVL}, \mathsf{DL}\}, \{\mathsf{MVR}, \mathsf{MVL}, \mathsf{CL}\}\}$ *and all* $\mathsf{Y} \in \{\lambda, \mathsf{scf}, \mathsf{scf\text{-}cscpp}\}$, *the following holds:*

$$\mathcal{L}(\text{det-mon-Y-X}) = \mathsf{LRR}.$$

- No Restart-steps
- the language class LRR is robust – characterized by automata both with and without Complete Strong Correctness Preserving Property

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
    - immediate constituents correspond to reductions and the final irreducible sentence.

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
  - immediate constituents correspond to reductions and the final irreducible sentence.
  - det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata have this ability, too.

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
  - immediate constituents correspond to reductions and the final irreducible sentence.
  - det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata have this ability, too.
- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for the complement of any LRR-language.

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
  - immediate constituents correspond to reductions and the final irreducible sentence.
  - det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata have this ability, too.
- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for the complement of any LRR-language.
  - Again, this also holds for det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata.

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
  - immediate constituents correspond to reductions and the final irreducible sentence.
  - det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata have this ability, too.
- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for the complement of any LRR-language.
  - Again, this also holds for det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata.
  - Localization of syntactical errors and for syntactic error recovery.

Further research:

Conclusions

Why?

- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for LRR-languages:
    - immediate constituents correspond to reductions and the final irreducible sentence.
    - det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata have this ability, too.
- det-mon-*RLWC*-automata in strong cyclic form ensure a deterministic analysis by reduction for the complement of any LRR-language.
    - Again, this also holds for det-mon-cscpp-scf-$\{MVR, MVL, CL\}$-automata.
    - Localization of syntactical errors and for syntactic error recovery.

Further research:

- To study det-mon-RLWC-automata in strong cyclic form having minimal look-ahead window and minimal reductions for a given LRR-language.

Thank you for your attention!