

Ján Borsík, 1955 - 2019

2010



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My lecture is dedicated to the memory of Professor Ján Borsík.

You can find more information about the scientific work of Professor Ján Borsík in our joint paper:

L. Holá, R. Frič, Ján Borsík, 1955 - 2019, Tatra Mt. Math. Publ. 74 (2019), 1–6

Ján Borsík was born on 11. 9. 1955 in Trstené, Slovakia.

After completing his secondary education, in the period 1975 - 1980 he studied at the Faculty of Natural Sciences, Comenius University in Bratislava, Slovakia.

Already as a student, he participated in a scientific seminar devoted to real functions led by Professor Tibor Šalát and remained faithful to real functions all his life.

After his graduation, Janko Borsík obtained his RNDr. title in 1981 and, under the supervision of Professor Šalát, the scientific degree CSc. (equivalent to PhD).

Finally, based on his Habilitationsschrift, in 1999 he was promoted to Docent in Mathematics.

Since 1980 until his premature death, he was affiliated with the Mathematical Institute of Slovak Academy, Extension in Košice, as a Senior Scientific Fellow.

For a long time Professor Ján Borsík lectured at the Šafárik University and the Technical University in Košice and at the Prešov University in Prešov,

where his colleagues and students very much appreciated his scientific and pedagogical masterships. The same can be said about several PhD students and members of the broader Slovak mathematical community.

Above all, he took over from Professors Šalát and Neubrunn the responsibilities related to the ISCRF.

Since 1994 he managed biennial conferences first in Liptovský Ján and then in Stará Lesná, on comfortable premises of Academia, the Congress Centre of Slovak Academy of Sciences.

The 32nd ISCRF in Stará Lesná was the last one that Ján organized and managed.

The conferences resulted in a cooperation with the Polish "real functions community" and since 2004 ISCRF is annually held, alternatively on the Polish and Slovak side of High Tatras. TMMP serves as the journal in which participants publish their contributions.

Scientific interests of Ján Borsík covered a wider range, but his real interest and research concentrated on real analysis. In papers of Ján Borsík mostly topological methods and measure theory methods are used. His scientific direction was influenced by his supervisor Professor T. Šalát.

The first paper of Ján is a joint paper with Jozef Doboš:

J. Borsík, J. Doboš, On functions whose composition with every metric is a metric, (in Russian), Math. Slovaca 31 (1981), 3-12.

The results of this paper were used in further joint papers with J. Doboš.

The paper J. Borsík, J. Doboš, On a product of metric spaces, Math. Slovaca 31 (1981), 193-205, is by Google Scholar Citations the most cited paper of Ján Borsík, it has 71 records.

The paper J. Borsík, J. Doboš, On metric preserving functions, Real Anal. Exchange 13 (1987-88), 285–294 has 44 records by Google Scholar Citations.

Ján Borsík also studied functions with closed graphs, functions that preserve cauchy sequences and cauchy nets, functions that preserve convergence of infinite series, oscillations for quasicontinuity, almost continuity, convergences of functions and generalized continuities.

Most of the research of Ján Borsík is devoted to the study of quasicontinuity and its weaker and stronger forms. The notion of quasicontinuity is a classical notion which was introduced by Kempisty in 1932 for real functions of real variables and later generalized for topological spaces by many authors, for example Professor Neubrunn. Perhaps the first mention of the condition of quasicontinuity can be found in the paper of R. Baire in the study of continuity points of separately continuous functions from $\mathbb{R}^2$ into $\mathbb{R}$.

R. Baire, *Sur les fonctions des variables reelles*, Ann. Mat. Pura Appl. 3 (1899), 1–122.

S. Kempisty, *Sur les fonctions quasi-continues*, Fund. Math. 19 (1932), 184–197.

T. Neubrunn, *Quasi-continuity*, Real Anal. Exchange 14 (1988), 259–306.

Definition. Let X and Y be topological spaces and $f : X \rightarrow Y$. Let $x_0 \in X$. The function f is quasicontinuous at x_0 , if for every open neighborhood U of x_0 and for every open neighborhood V of $f(x_0)$ there is a nonempty open set $G \subset U$ such that $f(x) \in V$ for every $x \in G$. f is quasicontinuous if it is quasicontinuous at every point of X .

An equivalent definition of quasicontinuity can be found in the paper of Levine

[Le] N. Levine, Semi-open sets and semicontinuity in topological spaces, Amer. Math. Monthly 68 (1961), 44-46

We say that a subset A of X is semi-open if A is contained in the closure of its interior ($A \subset \overline{\text{Int}A}$). A function $f : X \rightarrow Y$ is quasicontinuous if and only if $f^{-1}(V)$ is semi-open for every open set $V \subset Y$.

We mention some results of Ján which we used in our research.

Lemma

Let X be a metrizable space. Let F be a closed nowhere dense subset of X . Then there is a continuous function $f : X \setminus F \rightarrow [0, 1]$ such that in every neighbourhood V of $z \in F$ there are $x, y \in V \setminus F$ with $f(x) = 0$ and $f(y) = 1$.

J. Borsík, *Points of continuity, quasicontinuity and cliquishness*, Rend. Ist. Math. Univ. Trieste 26 (1994), 5–20.

Theorem

Let X be a Baire metrizable space. Then the set $M \subset X$ is the set of all discontinuity points of some quasicontinuous function $f : X \rightarrow \mathbb{R}$ if and only if M is a F_σ set of the first Baire category.

The question to characterize the set of discontinuity points of real-valued quasicontinuous functions defined on a topological space X is posed in the paper

T. Šalát, *On discontinuity points of functions of some classes*, Acta Math. Univ. Comenian. 42-43 (1983), 121–124.

For $X = \mathbb{R}^2$ it was solved in

E. Grande, *Sur un probleme concernat les fontion quasicontinues*, Math. Slovaca 32 (1982), 309–312.

Theorem

Let X be an uncountable Polish space. Then there are 2^c quasi-continuous non Borel measurable functions from X to $[0,1]$.

Ľubica Holá, There are 2^c Quasicontinuous Non Borel Functions on Uncountable Polish Space, Results in Mathematics 76(3), 2021

Ľubica Holá, Dušan Holý and Warren Moors, Usco and Quasicontinuous mappings, De Gruyter, Studies in Mathematics, Volume 81, 2021

Theorem

Let X be an uncountable Polish space. Then for the topology of uniform convergence on compacta on the space $Q(X, \mathbb{R})$, of quasicontinuous functions from X to $\mathbb{R}$, we have

$$c(Q(X, \mathbb{R})) = d(Q(X, \mathbb{R})) = e(Q(X, \mathbb{R})) = L(Q(X, \mathbb{R})) = \\ s(Q(X, \mathbb{R})) = nw(Q(X, \mathbb{R})) = \pi w(Q(X, \mathbb{R})) = w(Q(X, \mathbb{R})) = 2^c.$$

J. Borsík, L. Holá, D. Holý, Baire spaces and quasicontinuous mappings, Filomat 25 (2011), 69–83

Theorem

Let X be a Baire space. Let $\{f_n : n \in \omega\}$ be a sequence of real-valued quasicontinuous functions defined on X pointwise convergent to a function $f : X \rightarrow \mathbb{R}$. Then the following are equivalent:

- (1) f is quasicontinuous;*
- (2) $\{f_n : n \in \omega\}$ is equi-quasicontinuous.*

Quasicontinuity and pointwise convergence

Definition

Let $\{f_n : n \in \omega\}$ be a sequence of real-valued functions defined on a topological space X . We say that the sequence $\{f_n : n \in \omega\}$ is equi-quasicontinuous at $x \in X$ if for every $\epsilon > 0$ and every open neighbourhood U of x there is $n_0 \in \omega$ and a nonempty open set $W \subset U$ such that $|f_n(z) - f_n(x)| < \epsilon$ for every $z \in W$ and for every $n \geq n_0$. We say that $\{f_n : n \in \omega\}$ is equi-quasicontinuous if it is equi-quasicontinuous at every $x \in X$.

L'. Holá, D. Holý, Pointwise convergence of quasicontinuous mappings and Baire spaces, Rocky Mountain Math. J. 41 (2011), 1883–1894

List of publications of Ján Borsík I

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2000



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