# 40th International Summer Conference on Real Functions Theory

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# Preface

40th International Summer Conference on Real Functions Theory has been organized by the Mathematical Institute of the Slovak Academy of Sciences following the tradition in conferences on real functions theory dating back to 1971 when it was founded by professor Tibor Šalát and professor Pavel Kostyrko.

This booklet contains the list of participants, the abstracts and the list of authors. Participants are invited to send their contributions to the journal

**Tatra Mountains Mathematical Publications**.

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# Abstracts

## Conditional Cauchy equations for set-valued maps

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Let $(X,+)$ be a group, $(Y,+)$ be a commutative monoid, and $K$ be a submonoid of $Y$. Let $=_K$ mean the equivalence relation

$$A =_K B \iff A + K = B + K$$

for $A, B \in n(Y) := 2^Y \setminus \{\emptyset\}$. We consider two conditional Cauchy equations,

$$\forall\, x, y \in X \quad \big[F(x) + F(y) \neq_K F(0) \implies F(x+y) =_K F(x) + F(y)\big],$$

$$\forall\, x, y \in X \quad \big[F(x+y) \neq_K F(0) \implies F(x+y) =_K F(x) + F(y)\big],$$

in the class of set–valued maps $F\colon X \to n(Y)$. We check if such mappings have to be $K$–additive (the notion of $K$–additivity was introduced in [1]).

### References:

[1] Jabłońska, E. and Jabłoński, W.: Properties of K-additive set-valued maps, *Results Math.* 78:221 (2023).

# Ultrafilters and Ramsey theory

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In this presentation, we explore the interplay between Ramsey theory, ultrafilters, and topological algebra, highlighting ultrafilters as a powerful tool for discovering monochromatic patterns. Motivated by the classical Schur theorem, we will introduce Schur ultrafilters, discuss their set-theoretic and combinatorial properties, and demonstrate several applications. In particular, we will show how Schur ultrafilters describe Bohr compactifications of topological groups and provide a characterization for the automatic continuity of group operations in chart groups. Finally, using ultrafilter methods, we present extensions of the classical Hindman's and Hales-Jewett theorems.

**Keywords:** Ramsey theory, Schur ultrafilter, Bohr compactification.

# Separable non-negativity and equations for pairs of functions on big sets

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In our joint results with Rayene Menzer, presented at the XXXVIII. ISCRFT (in 2024), we considered functions $f\colon \mathbb{R}^m \to \mathbb{C}$ and $g\colon \mathbb{R}^k \to \mathbb{C}$ satisfying the condition

$$f(x)g(y) = 0 \tag{1}$$

for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{m+k}$ is large in some sense (it has a positive $m+k$ dimensional Lebesgue measure or it is a second category Baire set). We proved that $f$ or $g$ vanishes on a large subset of $\mathbb{R}^m$ or $\mathbb{R}^k$, respectively, in the same sense. If, in addition, we assume that the functions $f$ and $g$ are generalized polynomials, we obtain that one of them is identically equal to zero.

These investigations have been modified into 3 different directions.

Based on the arguments in [2], we investigated non-negativity of separable expressions on big subsets of product spaces in the following sense. Let $k$, $m \in \mathbb{N}$. Suppose that $f\colon \mathbb{R}^k \to \mathbb{R}$ and $g\colon \mathbb{R}^m \to \mathbb{R}$ satisfy

$$f(x)g(y) \geq 0\,, \tag{2}$$

for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{k+m}$ has a positive $k + m$ dimensional Lebesgue measure or $D$ is a second category Baire set. Then either one of these functions vanishes on a big set (in the same sense) or both $f$ and $g$ keep their signs on sets of positive ($k$ and $m$ dimensional) Lebesgue measure or on second category Baire sets, respectively. In the particular case when $k = m = 1$ and $f$ and $g$ are generalized monomials of even degrees, we conclude that (2) holds for every $(x, y) \in \mathbb{R}^2$.

Balcerzak and Popławski [1] considered a real generalized polynomial $f$ fulfilling $f(x)f(y) = 0$ for all $(x, y) \in D$, where $D \subseteq \mathbb{R} \times \mathbb{R}$ is a Borel set which is non-meager in a mixed sense, i.e., it does not belong to the Fubini product of the $\sigma$-ideals of Lebesgue zero sets and first category sets, respectively. They concluded that $f$ is identically equal to zero. Supposing that $f$ is a real generalized monomial fulfilling $f(x)f(y) \geq 0$ for all $(x, y) \in D$, where $D$ fulfills the previous assumptions, they concluded that $f$ preserves its sign on $\mathbb{R}$.

Motivated by a question of Peter Eliaš, we have investigated the functional equation

$$F(f(x), g(y)) = 0\,, \tag{3}$$

where $F$ is a given operation. Assuming that $S \subseteq \mathbb{C}\times\mathbb{C}$ and $F : S \to \mathbb{C}$ is locally cancellative in the sense that, for all $z \in \mathbb{C}$ and $w \in \mathbb{C}$, the equation $F(z, w) = 0$ has at most one solution in its first (resp., second) variable if the second (resp., first) variable is fixed, we can establish the following theorem: If $f : \mathbb{R}^k \to \mathbb{C}$ and $g : \mathbb{R}^m \to \mathbb{C}$ satisfy (3) for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{k+m}$ has a positive $k + m$ dimensional Lebesgue measure or $D$ is a second category Baire set, then $f$ and $g$ are constants on sets of positive ($k$ and $m$ dimensional) Lebesgue measure or on second category Baire sets, respectively.

**Keywords:** Lebesgue measure, second category Baire sets, alternative equations, generalized polynomials.

## References:

[1] M. Balcerzak, M. Popławski: Extending Theorems of Boros and Menzer, *Preprint arXiv:* 2603.25609, (2026).

[2] Z. Boros, R. Menzer: An alternative equation for generalized polynomials involving measure and category constraints, *Acta Math. Hungar.* 175 (2025), 376–394.

# Kuratowski convergence of Clarke subdifferentials

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The presentation focuses on two applications of Kuratowski convergence of graphs of multifunctions in the Euclidean space. Firstly, we generalise to Lipschitz functions the classical theorem stating that given a sequence of smooth functions with locally uniformly convergent derivatives, we obtain the local uniform convergence of the functions themselves (provided they were convergent at one point). Secondly, we present the reverse theorem for the squared distance function.

We know from the Rademacher Theorem that a locally Lipschitz function is differentiable almost everywhere, so we can define the subdifferential of a locally Lipschitz function $f : \Omega \to \mathbb{R}^k$, where $\Omega$ is an open set in $\mathbb{R}^m$ in the way Clarke did

$$\partial f(x) := conv\{\lim_{n\to+\infty} f'(x_n) | x_n \in \Omega \setminus Z, x_n \to x, \{f'(x_n)\}_{n=1}^{+\infty} \text{ is convergent}\}$$

where $Z$ is the set of the non-differentiability points of $f$.

Next, for a given metric space $(X, d)$ and a sequence of sets $A_n \subset X$ we define the Kuratowski lower and upper limits

$$\liminf_{n\to+\infty} A_n := \{x \in X \mid \exists x_n \in A_n \colon x_n \to x\},$$
$$\limsup_{n\to+\infty} A_n := \{x \in X \mid \exists n_1 < n_2 < \ldots, x_{n_k} \in A_{n_k} \colon x_{n_k} \to x\}.$$

If those two sets are equal to a set $A$ than we say the sequence $A_n$ tends in a Kuratowski sense to $A$.

We use the Kuratowski convergence of graph of multifunctions as a generalisation of local uniform convergence of functions.

We present a theorem stating that if a sequence of graphs of Clarke subdifferential of L-Lipschitz functions defined on a domain in $\mathbb{R}^m$ tends in the Kuratowski sense to a graph of a multfinunction which is univalent almost everywhere, then the sequence of those L-Lipschitz function is locally uniformly convergent (provided it was convergent at one point).

The second problem is the reverse of the first one for distance functions. Kuratowski convergence of a sequence of closed nonempty sets $A_n \subset \mathbb{R}^2$ to a nonempty set $A$ implies the local uniform convergence of the distance functions $dist(\cdot, A_n)$ to $dist(\cdot, A)$ and the Kuratowski convergence of graphs of $\partial dist^2(\cdot, A_n)$ to a graph of $\partial dist^2(\cdot, A)$, but does not imply the Kuratowski convergence of graphs of $\partial dist(\cdot, A_n)$.

Note that, the set of points where the function $dist^2(\cdot, A)$ is not differentiable, is the medial axis of the set A, which is used to study singularities of $A$.

**Keywords:** Clarke subdifferential, Kuratowski convergence, limit, Lipschitz function, multifunction, graph, distance function.

## References:

[1] L. Birbrair, M. Denkowski, Medial axis and singularities, *Journal of Geometric Analysis* 27 (2017), 2339–2380.
[2] F. Clarke, Generalized gradients and applications, *Trans. Amer. Math. Soc.* 205 (1975), 247-262.
[3] G. Lebourg, Generic differentiability of Lipschitzian functions. *Transactions of the American Mathematical Society*, Volume 256, (1979), 125-145.
[4] R. T. Rockafellar, R. Wets, *Variational analysis.* Springer Verlag, 1998.

# Cardinals Inspired by Convergence of Sequences of Functions

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Suppose we have two notions of convergence of sequences of functions, say $\alpha$ and $\beta$ (e.g. pointwise and uniform convergence). By $non(\alpha, \beta)$ we denote the smallest cardinality of an infinite topological space $X$ such that $\alpha$ and $\beta$ are indistinguishable in $\mathcal{C}(X)$ i.e. for every sequence of continuous functions $f_n : X \to \mathbb{R}$, the sequence $(f_n)$ is $\alpha$-convergent to $f \in \mathcal{C}(X)$ if and only if $(f_n)$ is $\beta$-convergent to $f$.

For instance, it is not difficult to show that

$$non(\text{pointwise}, \text{uniform}) = \aleph_0,$$

and Bukovský, Recław and Repický [2] proved that

$$non(\text{pointwise}, \text{quasi-normal}) = \mathfrak{b},$$

where $\mathfrak{b}$ is the bounding number and the quasi-normal convergence was introduced by Császár and Laczkovich [3] (they used the name equal convergence, whereas the name quasi-normal convergence was coined by Bukovská [1]).

In this talk, I will discuss $non(\alpha, \beta)$ with $\alpha$ and $\beta$ being the ideal versions of pointwise and quasi-normal convergence, and, time permitting, comment on the case of other kinds of ideal convergences. This is joint work with Adam Kwela [4, 5, 6].

## References:

[1] Z. Bukovská, Quasinormal convergence, *Math. Slovaca* 41 (1991), no. 2, 137–146

[2] L. Bukovský, I. Recław, and M. Repický, Spaces not distinguishing pointwise and quasinormal convergence of real functions, *Topology Appl.* 41 (1991), no. 1-2, 25–40. MR 1129696

[3] Á. Császár and M. Laczkovich, Some remarks on discrete Baire classes, *Acta Math. Acad. Sci. Hungar.* 33 (1979), no. 1-2, 51–70.

[4] R. Filipów, A. Kwela, Yet another ideal version of the bounding number, *J. Symb. Log.* 87 (2022), no. 3, 1065–1092.

[5] R. Filipów, A. Kwela, Spaces not distinguishing ideal pointwise and $\sigma$-uniform convergence, *Ann. Pure Appl. Logic* 176 (2025), no. 9, Paper No. 103609.

[6] R. Filipów, A. Kwela, Two $\mathfrak{b}$ or not two $\mathfrak{b}$?, *J. Symb. Log.* 2026:1-15, published online.

# On some generalized topologies satisfying all separation axioms

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The presentation based on the papers:

1. Hejduk, J., Loranty, A.: *On abstract generalized topological spaces generated by the density type operators*, Hacet. J. Math. Stat. 49(6), 1907–1914 (2020).

2. Hejduk, J., Loranty, A.: *On some Generalized Topologies Satisfying all Separation Axioms*, Results Math., 79, 38 (2024).

Let $\langle X, \mathcal{S}, \mathcal{J}\rangle$ be a measurable space where $\mathcal{S}$ is a $\sigma$-algebra of subsets of $X$ and $\mathcal{J} \subset \mathcal{S}$ is an admissible $\sigma$-ideal. Let $\Phi : \mathcal{S} \to 2^X$ be an operator satisfying the following properties:

1° $\Phi(\emptyset) = \emptyset$, $\Phi(X) = X$;

2° for any $A, B \in \mathcal{S}$ if $A \subset B$ then $\Phi(A) \subset \Phi(B)$;

3° for any $A, B \in \mathcal{S}$ if $A \triangle B \in \mathcal{J}$ then $\Phi(A) = \Phi(B)$;

4° $A \setminus \Phi(A) \in \mathcal{J}$ for any $A \in \mathcal{S}$.

If $\langle X, \mathcal{S}, \mathcal{J}\rangle$ has a hull property then we can extand the operator $\Phi$ putting for any $A \subset X$

$$\Phi_h^*(A) = \Phi(B),$$

where $B$ is an $\mathcal{S}$-measurable hull of $A$. Then

$$\Phi_h^* : 2^X \to 2^X.$$

**Theorem 1.** The family

$$\mathcal{T}_h^* = \{A \subset X : A \subset \Phi_h^*(A)\}$$

is a strong generalized topology and is not equal to $2^X$.

We shall say that a space $\langle X, \mathcal{S}, \mathcal{J}\rangle$ has a non-separation property with respect to $\mathcal{S}$ iff for any set $E \notin \mathcal{J}$ there exist disjoint subsets of $E$ which cannot be separated by set from $\mathcal{S}$.

Assume that $\langle X, \mathcal{S}, \mathcal{J}\rangle$ has a non-separation property with respect to $\mathcal{S}$ and satisfies C.C.C. The space $\langle X, \mathcal{T}_h^*\rangle$ is

- a Hausdorff space;
- a hereditarity normal space;
- a normal space;
- a perfectly normal space;
- every set $A \subset X$ is $\mathcal{T}_h^*$-$G_\delta$-set.

The presentation contains numerous examples of the operator $\Phi_h^*$ based on the outer Lebesgue measure.

**Keywords:** generalized topology, density type operator, separation axioms.

## References:

[1] Császár, Á.: Generalized open sets. *Acta Math. Hung.* 75, 65—87 (1997).
[2] Császár, Á.: Separation axioms for generalized topologies. *Acta Math. Hung.* 104(1–2), 63—69 (2004).
[3] Grzegorek, E., Labuda, I.: On two theorems of Sierpiński. *Arch. Math.* 110, 637-–644 (2018).
[4] Grzegorek, E., Labuda, I.: Partitions into thin sets and forgotten theorems of Kunugi and Luzin–Novikov. *Colloq. Math.* 155, 267—285 (2019).
[5] Hejduk, J., Loranty, A.: On abstract generalized topological spaces generated by the density type operators. *Hacet. J. Math. Stat.* 49(6), 1907—1914 (2020).
[6] Hejduk, J., Loranty, A.: On some Generalized Topologies Satisfying all Separation Axioms, *Results Math.* 79, 38 (2024).
[7] Wilczyński, W.: A generalization of the density topology. *Real. Anal. Exch.* 32(2), 273—286 (2006/2007).

# Ján Borsík, 1955 – 2019

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My lecture is dedicated to the memory of Professor Ján Borsík. More information about the scientific work of Professor Ján Borsík can be found in our joint paper with Professor Roman Frič [1].

## References:

[1] Ľ. Holá, R. Frič: Ján Borsík, 1955 – 2019, *Tatra Mt. Math. Publ.* 74 (2019), 1–6.

# On $K$-measurable set-valued maps

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Inspired by the paper of Himmelberg [2], working with Judyta Bąk, Eliza Jabłońska, Małgorzata Terepeta, we generalize definitions of [weakly] measurable set-valued maps.

Let $(X, \mathcal{A})$ be a measurable space, $Y$ be a real topological vector space and $K \subset Y$ be a convex cone. An s.v. map $F\colon X \to 2^Y \setminus \{\emptyset\}$ is called:

- *weakly $K$-$\mathcal{A}$-measurable*, if $F^-(W - K) \in \mathcal{A}$ for every nonempty open set $W \subset Y$,
- *$K$-$\mathcal{A}$-measurable*, if $F^+(W + K) \in \mathcal{A}$ for every nonempty open set $W \subset Y$,

where

$$F^-(S) := \{x \in X\colon F(x) \cap S \neq \emptyset\},$$
$$F^+(S) := \{x \in X\colon F(x) \subset S\}$$

for $S \subset Y$.

We study properties of such maps. We apply the notion of [weak] $K$-measurability to generalize some classical results of Nikodem [5] on $K$-continuity of measurable $K$-midconvex and $K$-midconcave set-valued maps.

**Keywords:** K-continuity, K-measurability, K-midconvex map, K-midconcave map, set-valued map.

## References:

[1] J-P. Aubin, H. Frankowska, *Set-Valued Analysis*, Birkhäuser, Boston-Basel-Berlin, 1990.
[2] C.J. Himmelberg, Measurable relations, *Fund. Math.* 87 (1975), 53–72.
[3] G. Horbaczewska, E. Jabłońska, W. Jabłoński, M. Terepeta, On multiadditive set-valued maps "modulo K". *Aequat. Math.* 99 (2025), 2609–2629.
[4] K. Kuratowski, *Topologie*, Instytut Matematyczny PAN, Warszawa–Lwów, 1933.
[5] K. Nikodem, K–convex and K–concave set–valued functions, *Zeszyty Nauk. Politechniki Łódzkiej Mat.* 559; *Rozprawy Mat.* 114, Łódź 1989.

# On the optimized certainty equivalent

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Assume that $\mathcal{X}$ is a family of all bounded random variables on a given probability space. Let $\mathcal{U}$ be a family of all non-decreasing concave functions such that $u(0) = 0$ and $u(x) \leq x$ for $x \in \mathbb{R}$. For any $u \in \mathcal{U}$, the optimized certainty equivalent of $X \in \mathcal{X}$ is defined by (cf. [1])

$$S_u(X) = \sup\,\{t + E[u(X - t)] : t \in \mathbb{R}\}.$$

We investigate the problem of recovering of utility function from its optimized certainty equivalent. Moreover, we present some applications of our results.

**Keywords:** optimized certainty equivalent, utility function, representation, risk measure.

## References:

[1] Ben-Tal A., Teboulle M.: Expected utility, penalty functions, and duality in stochastic nonlinear programming, *Management Sci.* 32 (1986), 1445–1466.

# Sequence spaces of Baire-one functions and their porosity


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Let $(X, \varrho)$ be a compact metric space and let $B_1(X, X)$ denote the collection of Baire class-one self-maps on $X$. As in [2], a non-autonomous $B_1$-dynamical system, or $B_1$-NDS, is defined as a sequence of $B_1(X, X)$. Let $\mathcal{F}$ be the family of all such systems. Then for any $F = (f_n)_{n\in\mathbb{N}} \in \mathcal{F}$ and $G = (g_n)_{n\in\mathbb{N}} \in \mathcal{F}$, we define a metric by

$$\varrho_{\mathcal{F}}(F, G) = \sup_{n\in\mathbb{N}} \varrho_u(f_n, g_n),$$

where $\varrho_u$ is the uniform metric on $B_1(X, X)$ (that is, $\varrho_u(f, g) := \sup_{x\in X} \varrho(f(x), g(x))$ for $f, g \colon X \to X$).

Now let $X = I$ where $I$ is the unit interval $[0, 1]$. A. Alikhani-Koopaei in [2] considered the following subsystems of $\mathcal{F}$:

- $\mathcal{F}_u(I)$ – the family of all uniformly convergent sequences;
- $\mathcal{F}_{eq}(I)$ – the family of pointwise convergent equi-$B_1$ sequences; i.e., systems $(f_n)_{n\in\mathbb{N}}$ where for every $\varepsilon > 0$, there exists a mapping (a gauge) $\delta \colon I \to \mathbb{R}^+$ such that for all $n \in \mathbb{N}$, the condition $\varrho(x, y) < \min\{\delta(x), \delta(y)\}$ implies $\varrho(f_n(x), f_n(y)) < \varepsilon$ for any $x, y \in I$;
- $\mathcal{F}_{p.w.}(I)$ – the family of all Baire-one sequences that converge pointwise to a function $f \in B_1(I, I)$;
- $\mathcal{F}(I)$ – the family of all Baire-one sequences that converge pointwise.

It is known that (see [1, 2])

$$\mathcal{F}_u(I) \subset \mathcal{F}_{eq}(I) \subset \mathcal{F}_{p.w.}(I) \subset \mathcal{F}(I). \tag{4}$$

In [1], it is proved that the first three families of sequences are closed subsets of the fourth. In [2], it is also shown that the first family is nowhere dense in the second, and the second is so in the third.

We prove that each such family is not only nowhere dense, but also is lower 2/3-porous in the next one.

If one modifies these families where sequences of Baire-one functions are real-valued uniformly bounded, then they are Banach spaces and each space of the respective chain is c-porous in the next one.

**Keywords:** sequence spaces, complete spaces, lower porosity, c-porosity.

**References:**

[1] A. Alikhani-Koopaei, On dynamics of families of equi-Baire one functions on metric spaces, Topology Appl. 322 (2022), 108267.

[2] A. Alikhani-Koopaei, On dynamics of convergent sequences of functions on metric spaces, Topology Appl. 305 (2026), 109849.

# Properties "modulo $K$" of set-valued maps

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The starting point of our considerations is Nikodem's dissertation [7], where the notions of $K$-boundedness, $K$-continuity, and $K$-midconvexity were introduced.

Following Nikodem, if $X, Y$ are real topological vector spaces and $K \subset Y$ is a convex cone, a set-valued map $F\colon X \to 2^Y \setminus \{\emptyset\}$ is called *$K$-continuous at* $x_0 \in X$, if for every neighborhood $W \subset Y$ of 0 there is a neighborhood $U$ of $x_0$ such that

$$F(x) \subset F(x_0)+W+K \quad \text{and} \quad F(x_0) \subset F(x)+W+K \quad \text{for all } x \in U.$$

We give some properties of $K$-continuous maps; among others, we compare the notion of $K$-continuity introduced by Nikodem with the natural notion of continuity introduced by Kuratowski. Finally, we apply this notion to $K$-additive set-valued maps, i.e., $F\colon X \to 2^Y \setminus \{\emptyset\}$ such that

$$F(x+y) \subset F(x)+F(y)+K \text{ and } F(x)+F(y) \subset F(x+y)+K \text{ for } x, y \in X,$$

and (if time allows) to other special set-valued maps.

## References:

[1] E. Jabłońska, W. Jabłoński, Properties of K-additive set-valued maps, *Results Math.* (2023) 78:221.

[2] E. Jabłońska, Characterization of continuous additive set-valued maps "modulo K" on finite dimensional linear spaces, *Math. Slovaca* 74 (2024), 1165–1172.

[3] E. Jabłońska, On almost everywhere K-additive set-valued maps, *Ann. Math. Sil.* 38 (2024), 29–36.

[4] E. Jabłońska, On set-valued derivations modulo K, *J. Convex Anal.* 32 (2025), 1135–1144.

[5] E. Jabłońska, On K-sublinear set-valued maps. *Period. Math. Hungar.* 91 (2025), 452–460.

[6] G. Horbaczewska, E. Jabłońska, W. Jabłoński, M. Terepeta, On multiadditive set-valued maps "modulo K", *Aequationes Math.* 99 (2025), 2609–2629.

[7] K. Nikodem, K-convex and K-concave set-valued functions. *Zeszyty Nauk. Politechniki Łódzkiej Mat.* 559, *Rozprawy Mat.* 114, Łódź 1989.

# The Fréchet derivative is in the first Baire class

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Let $X$ be a normed linear space, $Y$ a Banach space, $G \subset X$ an open set and $f : G \to Y$ a mapping. In [1] we show that the Fréchet derivative $f'$ of $f$ is of the first Baire class on the (possibly empty) set $D \subset G$ where it is defined.

**Keywords:** Fréchet derivative, first Baire class.

## References:

[1] E. Kopecká, L. Zajíček, The Fréchet derivative is in the first Baire class, *arXiv*:2608.02233

# Cardinal characteristics of selected ideals on countable sets

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In this talk, I will study several cardinal characteristics associated with a family of ideals $\mathsf{conv}_\alpha$ on a countable set, indexed by countable ordinals $\alpha$. These ideals arise naturally from the characterization of compact countable spaces homeomorphic to $\omega^\alpha \cdot n + 1$ with the order topology, in terms of the existence of convergent subsequences defined on sets not belonging to $\mathsf{conv}_\alpha$.

I will then discuss the structural properties of the family $\mathsf{conv}_\alpha$ in the Katětov order. For limit ordinals $\alpha$, I will present the associated ideals $\mathsf{conv}_{<\alpha}$, which serve as greatest lower bounds of the preceding $\mathsf{conv}_\beta$, leading to intertwined decreasing hierarchies of ideals with different descriptive complexities.

This talk is based on [1] and [2].

## References:

[1] R. Filipów, M. Kowalczuk and A. Kwela, Critical ideals for countable compact spaces, *Fund. Math.* 274 (2026), no. 1, 1–23.

[2] M. Kowalczuk, Infima and cardinal characteristics of critical ideals for countable compact spaces, *arXiv*:2603.00674

# $\mathfrak{c}$-lineability and $\mathfrak{c}$-spaceability of some families of real functions

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We would like to present some results about $\mathfrak{c}$-lineability and $\mathfrak{c}$-spaceability of some families $\mathcal{F}$ of real functions defined on an interval $I$. The main goal is to formulate general conditions under which any non-empty family $\mathcal{F} \subset \mathbb{R}^I$ of functions is $\mathfrak{c}$-spaceable or $\mathfrak{c}$-lineable. Generally, we consider the families of function of the form $\mathcal{F} = \mathcal{F}_1 \setminus \mathcal{F}_2$. In most cases, families of functions for which lineability and spaceability are studied have such a form. Most often, family $\mathcal{F}_2$ is seemingly "very close" to $\mathcal{F}_1$ or consists of "almost all" functions. The results obtained by us are a generalization of previous ideas, from [1].

The main idea of our constructions is to "reproduce" one function to obtain $\mathfrak{c}$-dimensional (closed) linear space. For this "reproduction" we use the Fichtenholz-Kantorovich Theorem, applied to a countable family of pairwise disjoint intervals contained in the domain of functions from the considered class. The initial function is "squashed" and "pasted" into disjoint intervals included in the domain of constructed function.

We present some natural conditions under which families constructed in this way belongs to considered class (it always forms $\mathfrak{c}$ dimensional linear space). Finally, we present numerous examples of applications of the presented theory.

Presented results was obtained jointly with Małgorzata Turowska.

**Keywords:** $\mathfrak{c}$-lineability, $\mathfrak{c}$-spaceability.

## References:

[1] A. Bartoszewicz, M. Filipczak, M. Terepeta, Lineability of Linearly Sensitive Functions, *Results Math.* (2020) 75:64.

[2] S. Kowalczyk, M. Turowska, On c-lineability, *J. Math. Anal. Appl.* 543 (2025), 128916, DOI:10.1016/j.jmaa.2024.128916

# Infinite convolutions of measures applied to achievement sets

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Let $E(\Sigma, q) := \left\{\sum_{i=1}^{\infty} \sigma_i q^i \colon (\sigma_i) \in \Sigma^{\mathbb{N}}\right\}$ for a finite set $\Sigma \subset \mathbb{R}$ and $q \in (0,1)$. Sets of this form are a generalisation of achievement sets of the so-called multigeometric series. We associate with $E(\Sigma, q)$ a Borel measure $\mu_\infty$ that is a weak limit of convolutions $\mu_n = \nu_1 * \cdots * \nu_n$, where $\nu_n(B) = \frac{|B \cap (q^n \Sigma)|}{|\Sigma|}$. We prove several new equivalent conditions for $E\left(\Sigma, 1/|\Sigma|\right)$ to contain an interval, see [1, Theorem 8], one of which is the following:

$$\exists_{\delta>0} \forall_{n\in\mathbb{N}} \exists_{k\in\mathbb{N}} \sum_{\sigma\in\Sigma} \exp\left(\frac{2\pi i n \sigma}{\delta |\Sigma|^k}\right) = 0.$$

This way we generalise results of Kenyon [2], who characterised sets $E\left(\{0,1,u\}, 1/3\right)$, and Nitecki [3], who proved that the condition for $\Sigma$ to contain integer numbers of precisely all remainders modulo $|\Sigma|$, is sufficient for $E(\Sigma, 1/|\Sigma|)$ to contain an interval. We obtain some intermediate results concerning the measure $\mu_\infty$, e.g. $E = E(\Sigma, 1/|\Sigma|)$ contains an interval if and only if $\mu_\infty$ is absolutely continuous with respect to Lebesgue measure $\lambda$, and in that case $\mu_\infty(B) = \frac{\lambda(B\cap E)}{\lambda(E)}$.

**Keywords:** self-similar set, achievement set, infinite convolution of measures.

## References:

[1] S. Głąb, M. Kula, Kenyon theorem revisited, *arXiv*:2508.19454v1 [math.GN], 2025.

[2] R. Kenyon, Projecting the one-dimensional Sierpiński gasket, *Israel J. Math.* 97, 221–238, 1997.

[3] Z. Nitecki, Cantorvals and subsum sets of null sequences, *Amer. Math. Monthly* 122, no.9, 862–870, 2015.

# Interrelationships among axioms for measures of noncompactness

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We investigate the independence of the axioms in the Banaś–Goebel approach to the theory of measures of noncompactness in Banach spaces. This results in weakening the requirements imposed in the original system of the axioms. We show that in the case of four axioms (out of the totality of five axioms) they turn out independent of the remaining ones. For the troublemaking axiom, we provide a full solution in finite-dimensional spaces and a partial one in the infinite-dimensional case.

The presented results were co-authored by Jerzy Grzybowski and Piotr Kasprzak.

**Keywords:** axiomatic measures of noncompactness, Banach spaces, independence of axioms.

## References:

[1] J. Grzybowski, P. Kasprzak, P. Maćkowiak, On the independence of axioms for measures of non-compactness, 2026.

# Endpoint weighted bounds for rough maximal singular integrals

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The study of singular integral operators, initiated by Calderón and Zygmund in the 1950s, remains a central theme in Fourier analysis.

Given a function $\Omega$ on $\mathbb{S}^{d-1}$, one defines the singular integral operator

$$T_\Omega f(x) = \text{p.v.} \int_{\mathbb{R}^d} f(x-y)\, \Omega\left(\frac{y}{|y|}\right) |y|^{-d}\, dy.$$

When $\Omega$ is smooth $T_\Omega$ belongs to the classical Calderón–Zygmund theory; in the absence of smoothness, it is referred to as a rough singular integral. Calderón and Zygmund proved that if $\Omega \in L \log L$ has zero mean, then $T_\Omega$ is bounded on $L^p$ for $1 < p < \infty$. The end point $p = 1$ remained opened for almost 40 years and was eventually settled in full generality by Seeger in 1996.

To study pointwise convergence, one considers the associated maximal operator,

$$T_\Omega^* f(x) = \sup_{\epsilon > 0} \left| \int_{|y|>\epsilon} f(x-y)\, \Omega\left(\frac{y}{|y|}\right) |y|^{-d}\, dy \right|.$$

Duoandikoetxea and Rubio de Francia showed that $T_\Omega^*$ is bounded on $L^p$ for $1 < p < \infty$, while the weak $(1,1)$ bound remained open until very recently. This endpoint conjecture was resolved by Xudong Lai, improving earlier $L \log\log L$ results of Bhojak and Mohanty. In this talk, we will see the endpoint weighted estimates for $T_\Omega^*$.

**Keywords:** singular integrals, weighted estimates.

# Minimal usco maps and bornologies

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Minimal usco maps (multifunctions) play an important role in several areas of mathematics, especially in functional analysis, optimization theory, nonsmooth analysis, and selection theory. Given Hausdorff space $X$, we will explore cardinal invariants of the space $MU(X)$, the space of minimal usco maps on $X$ with real values equipped with topology of uniform convergence on a given bornology $\mathcal{B}$. We also present conditions under which the topologies of uniform convergence on bornologies for minimal usco maps behave like metric ones, in the sense that the weight, netweight, density, Lindelöf number, and cellularity all coincide for these topologies.

**Keywords:** minimal usco map, topology of uniform convergence on bornology, density, weight, netweight, cellularity, character, uniform extent.

# Cantorval as a set of subsums of a series - new sufficient condition and its consequences

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Given a nonincreasing sequence of positive numbers $(a_n)$ such that the series $\sum a_n$ converges, let $E(a_n)$ denote the set of all subsums of $\sum a_n$, called the *achievement set* of $(a_n)$. It is well known (see [7]) that such a set can be a finite union of closed intervals, a Cantor set, or a Cantorval. We say that a compact and perfect set $D \subset \mathbb{R}$ is a Cantorval if every endpoint of every gap of $D$ (that is, every connected component of $\mathbb{R} \setminus D$) is an accumulation point of both other gaps and nondegenerate connected components of $D$ (see [11]).

Finding conditions that guarantee an achievement set is either a Cantor set or a Cantorval has become one of the central problems in the study of achievement sets (see, e.g., [1–5,8]). Despite the considerable amount of research devoted to this topic, a complete characterization of when a given achievement set is a Cantor set or a Cantorval remains unknown.

Let $r_n = \sum_{i>n} a_i$. In his seminal paper [9], Kakeya proved that if $a_n \leq r_n$ for almost every $n \in \mathbb{N}$, then $E(a_n)$ is a finite union of closed intervals, whereas if $a_n > r_n$ for all but finitely many $n \in \mathbb{N}$, then $E(a_n)$ is a Cantor set. In [10], Marchwicki and Miska posed the following problem: given any infinite set $K \subset \mathbb{N}$ such that $\mathbb{N} \setminus K$ is also infinite, does there exist a sequence $(a_n)$ for which $E(a_n)$ is a Cantorval and $a_n > r_n \iff n \in K$?

We introduce a new sufficient condition for an achievement set to be a Cantorval. We also demonstrate how this condition can be used to construct new examples of achievable Cantorvals. In particular, we show that Kakeya's conditions cannot provide a finer classification of achievement sets than the one established in his original theorem, thereby answering the question posed by Marchwicki and Miska.

The talk is based on the results of [6].

**Keywords:** achievement sets, Cantorvals, Kakeya conditions.

## References:

[1] T. Banakh, A. Bartoszewicz, M. Filipczak, E. Szymonik, Topological and measure properties of some self-similar sets, *Topol. Methods Nonlinear Anal.* 46(2) (2015), 1013–1028.

[2] A. Bartoszewicz, S. Głąb, J. Marchwicki, Recovering a purely atomic finite measure from its range, *J. Math. Anal. Appl.* 467 (2018), 825-841.
[3] A. Bartoszewicz, M. Filipczak, F. Prus-Wiśniowski, Semi-fast convergent sequences and k-sums of central Cantor sets, *Eur. J. Math.* 6 (2020), 1523-1536.
[4] A. Bartoszewicz, M. Filipczak, E. Szymonik, Multigeometric sequences and Cantorvals, *Cent. Eur. J. Math.* 12(7) (2014), 1000–1007.
[5] T. Filipczak, P. Nowakowski, Conditions for the difference set of a central Cantor set to be a Cantorval, *Results Math.* 78 (2023), article no. 166.
[6] P. Nowakowski, On a new condition implying that an achievement set is a Cantorval and its applications, *arXiv*:2512.17761.
[7] J.A. Guthrie, J.E. Nymann, The topological structure of the set of subsums of an infinite series, *Colloq. Math.* 55 (1988), 323–327.
[8] R. Jones, Achievement sets of sequences, *Am. Math. Mon.* 118(6) (2011), 508–521.
[9] S. Kakeya, On the partial sums of an infinite series, *Tôhoku c. Rep.* 3 (1914), 159–164.
[10] J. Marchwicki, J. Miska, On Kakeya conditions for achievement sets, *Results in Math.* 76 (2021) article no. 181.
[11] P. Mendes, F. Oliveira, On the topological structure of the arithmetic sum of two Cantor sets, *Nonlinearity* 7 (1994), 329–343.

# Operation * in the Baire space

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We will work in the Baire space $\mathbb{Z}^\omega$ with the addition operation by coordinates. For a family $\mathcal{F} \subseteq \mathcal{P}(\mathbb{Z}^\omega)$ we define:

$$\mathcal{F}^* = \{A \subseteq \mathbb{Z}^\omega : \forall F \in \mathcal{F}\ A + F \neq \mathbb{Z}^\omega\},$$

where $A + F = \{a + f : a \in A,\ f \in F\}$. Operation * was defined by Seredyński in [4].

Let $I = \{I_n\}_{n\in\omega}$ be an interval partition of $\omega$. For $x \in \mathbb{Z}^\omega$, define

$$M(x, I) = \{y \in \mathbb{Z}^\omega : \forall^\infty n\ y{\restriction}I_n \neq x{\restriction}I_n\}.$$

We denote by $\mathcal{M}_-$ the $\sigma$-ideal generated by sets of this form. It was introduced and studied in [3].

We also consider the following $\sigma$-ideals:

- $\mathcal{SMZ}^+$ consisting of sets $X \subseteq \mathbb{Z}^\omega$ such that for every interval partition there exists $z \in \mathbb{Z}^\omega$ such that:

$$\forall x \in X \quad \exists^\infty n \quad x{\restriction}I_n = z{\restriction}I_n.$$

- $\mathcal{ED}$, generated by eventually different sets

$$E_f = \{x : \forall^\infty n\ x(n) \neq f(n)\},$$

- $\mathcal{IE}$, generated by infinitely equal sets

$$I_g = \{x : \exists^\infty n\ x(n) = g(n)\},$$

- $\mathcal{H}$, generated by sets

$$H_{I,f} = \{x : \forall^\infty n\ \exists m \in I_n\ x(m) = f(m)\},$$

- $\mathcal{G}$, consisting of sets $X \subseteq \mathbb{Z}^\omega$ such that for every interval partition $I$ there exists $z$ satisfying

$$\forall x \in X \quad \exists^\infty n \quad \forall m \in I_n\ x(m) \neq z(m).$$

It turns out that if we rewrite these definitions to the Cantor space $2^\omega$, we have $\mathcal{H} = \mathcal{M}_- = \mathcal{M}$ and $\mathcal{G} = \mathcal{SMZ}^+ = \mathcal{SMZ}$, where $\mathcal{M}$ is a $\sigma$-ideal of meager sets and $\mathcal{SMZ}$ is a $\sigma$-ideal of strong measure zero sets. It turns out that in the Baire space, we can separate some of these families in ZFC or by assuming $\mathrm{add}(\mathcal{M}) = \mathfrak{c}$.

Our main theorem states that $\mathcal{M}_-$ preserves all classical cardinal invariants of the meager ideal $\mathcal{M}$:

$$\mathrm{add}(\mathcal{M}_-) = \mathrm{add}(\mathcal{M}), \qquad \mathrm{cov}(\mathcal{M}_-) = \mathrm{cov}(\mathcal{M}),$$

$$\mathrm{non}(\mathcal{M}_-) = \mathrm{non}(\mathcal{M}), \qquad \mathrm{cof}(\mathcal{M}_-) = \mathrm{cof}(\mathcal{M}).$$

$$\mathrm{cov}_t(\mathcal{M}_-) = \mathrm{cov}(\mathcal{M}).$$

Galvin-Mycielski-Solovay Theorem [2] says that on the real line we have $\mathcal{M}^* = \mathcal{SMZ}$. It was proved by Wohofsky in [5] that the Galvin-Mycielski-Solovay theorem does not hold in the Baire space, but it works in the Cantor space [1]. We managed to prove that in the Baire space it holds:

$$\mathcal{M}_-^* = \mathcal{SMZ}^+.$$

We also proved the following relationships between other $\sigma$-ideals:

$$\mathcal{ED}^* = \mathcal{IE}, \qquad \mathcal{IE}^* = \mathcal{ED},$$

$$\mathcal{H}^* = \mathcal{G}.$$

**References:**


[1] Sz. Żeberski D. Perkowska. Star operation, microscopic sets and porous sets. `https://arxiv.org/abs/2510.19437` (2025).
[2] R. Solovay F. Galvin J. Mycielski. Strong measure zero sets. *Notices Amer. Math. Soc.* (1973).
[3] M. Michalski, R. Rałowski, and Sz. Żeberski. On algebraic sums, trees and ideals in the Baire space. *Archive for Mathematical Logic* (2025).
[4] W. Seredyński. Some operations related with translation. *Colloquium Mathematicum* (1989).
[5] W. Wohofsky. *Special sets of real numbers and variants of the Borel Conjecture.* PhD Thesis (2013).

# The 1914 Kakeya hypothesis

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The study of the topological nature of sets of all possible subsums of absolutely convergent real series began over 100 years ago with a paper by S. Kakeya [1] (see also [2]). He discovered that the set of subsums of a convergent infinite series of positive terms is always compact and perfect. Kakeya observed that there are at least two possibilities for such sets: they can be *multi-interval sets* (i.e., finite unions of closed bounded intervals), or Cantor sets.

Let $\sum x_n$ be a convergent series of positive terms and let $r_n := \sum_{k>n} x_k$ be the $n$-th remainder of the series. If $x_n > r_n$, then we say that the $n$-th Kakeya condition holds. The set of all indices satisfying the Kakeya condition will be denoted by $K(x_n)$. If $x_n \leq r_n$, then we say that the $n$-th reversed Kakeya condition holds. The set of all indices with reversed Kakeya condtions will be denoted by $RK(x_n)$. The achievement set (or the set of subsums) is defined by $E(x_n) := \{y \in \mathbb{R} : \exists A \subset \mathbb{N} \; y = \sum_{n \in A} x_n\}$.

Kakeya proved that $K(x_n)$ is finite if and only if $E(x_n)$ is a multi-interval set. Also, he showed that if $RK(x_n)$ is finite, then $E(x_n)$ is a Cantor set. Kakeya believed that $E(x_n)$ is a Cantor set if only $K(x_n)$ is an infinite set. In [2], he candidly wrote: *"That the relation $x_n \leq r_n$ fails only for an infinite number of values of $n$, seems to be the necessary and sufficient condition that the set $E(x_n)$ should be nowhere dense; but I have no proof of it."* This is the 1914 Kakeya hypothesis. Only in 1980 Weinstein and Shapiro announced an example of a summable sequence $(y_n)$ of positive terms such that $E(y_n)$ is neither a multi-interval set nor a Cantor set. Subsets of the real line homeomorphic to $E(y_n)$ are called Cantorvals. The powerful Guthrie-Nymann Classification Theorem from 1988 asserts that there are only four topological possibilities for achievement sets: finite sets, multi-interval sets, Cantor sets and Cantorvals.

Nevertheless, there is some truth to Kakeya's conjecture: for every infinite set $K \subset \mathbb{N}$ with infinite complement, there exists an absolutely convergent series $\sum x_n$ such that $K(x_n) = K$ and $E(x_n)$ is a Cantor set. This non-trivial result was proved five years ago by J. Marchwicki and P. Miska in [3]. They also investigated whether the Cantor set could be replaced by a Cantorval. This is by no means an easy question. In all known examples of achievable Cantorvals, at least half (in terms of asymptotic density) of the Kakeya conditions are reversed. Marchwicki and Miska were able to show that for any positive number $\alpha$ there is a sequence $(x_n)$ such that $E(x_n)$ is a Cantorval and the asymptotic density of $RK(x_n)$ equals to $\alpha$. They asked if it can happen for $\alpha = 0$. During my talk a positive answer to the question will be presented based on my joint work with Jolanta Ptak [4].

**Keywords:** Kakeya hypothesis, achievement set, Cantorval, asymptotic density.

**References:**

[1] S. Kakeya, On the partial sums of an infinite series, *Tôhoku Sc. Rep.* 3 (1915), 159–164.

[2] S. Kakeya, *On the set of partial sums of an infinite series, Proc. Tokyo Math.-Phys. Soc.*, 2nd series, 7(1914), 250-251, `https://doi.org/10.11429/ptmps1907.7.14_250`

[3] J. Marchwicki, J. Miska, On Kakeya conditions for achievement sets, *Results in Math.* (2021) 76:181; `https://doi.org./10.1007/s00025-021-01479-2`

[4] F. Prus-Wiśniowski, J. Ptak, Achievable Cantorvals almost without reversed Kakeya conditions, *arXiv*:2412.08768v1

# Topological nature of Formal Concept Analysis within the framework of multifunction theory

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Formal Concept Analysis (FCA), introduced by Ganter and Wille [1, 2], is a mathematical method for data analysis. The central idea of FCA is definability, which describes the relationships between the subsets $A \subset X$ of objects and the subsets $B \subset Y$ of attributes. A formal concept is represented by a pair $(A, B) \in \mathcal{P}(X)\times\mathcal{P}(Y)$ together with a pair $(\alpha, \beta)$ of derivation operators $\alpha : \mathcal{P}(X) \to \mathcal{P}(Y)$ and $\beta : \mathcal{P}(Y) \to \mathcal{P}(X)$, where $\alpha$ determines subset B and $\beta$ determines subset A i.e.,

$$B = \alpha(A) \text{ and } A = \beta(B),$$

such that the following conditions are satisfied

$$\alpha(\beta(\alpha(A))) = \alpha(A) \text{ and } \beta(\alpha(\beta(B))) = \beta(B).$$

In the literature, eight different versions of concepts have been studied.

The basic structure in FCA is a formal context $(X, Y, \mathcal{R})$, where $X$ is a set of objects, $Y$ is a set of attributes, and $\mathcal{R}$ is a binary relation between objects and attributes, where $(x, y) \in \mathcal{R}$ means the object $x$ possesses the attribute $y$.

In this investigation, we replace the relation $\mathcal{R}$ by the multifunction $F : X \to Y$, defined as
$F(x) = \{y : x\mathcal{R}y\}$. Under some nature assumptions, the multifunction $F$ generates three additional multifunctions:

1. $F_c : X \to Y$ defined by $F_c(x) = Y \setminus F(x)$,
2. $F^- : Y \to X$ defined by $F^-(y) = \{x \in X : y \in F(x)\}$,
3. $F_c^- : Y \to X$ defined by $F_c^-(y) = X \setminus F^-(y)$.

Using these four multifunctions, we determine two topologies in $X$ and two in $Y$ such that all eight classic forms of definition $B = \alpha(A)$ and $A = \beta(B)$ can be equivalently characterized in the topological form, namely

$$B = \alpha(\mathcal{O}(A)) \text{ and } A = \beta(\mathcal{O}^*(B)),$$

where $\mathcal{O}$, $\mathcal{O}^*$ are the interior or closure operator in the appropriate topology.

**Keywords:** Formal Concept, Multifunction, Definability.

**References:**

[1] R. Wille, Restructuring lattice theory: an approach based on hierarchies of concepts, in: *Ordered Sets*, Rival, I. (Ed.), Reidel, Dordrecht-Boston, (1982) 445-470.

[2] B. Ganter, R. Wille, *Formal Concept Analysis, Mathematical Foundations*, Springer, Berlin, 1999.

# The robust Orlicz premium principle under uncertainty

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Let $(\Omega, \mathcal{F})$ be a measurable space and $\mu_0 : \mathcal{F} \to [0, 1]$ be a capacity, that is a monotone set function such that $\mu_0(\emptyset) = 0$ and $\mu_0(\Omega) = 1$. The robust Orlicz premium principle under uncertainty for the risk $X$ is defined through the equation

$$H_{\alpha,S,\Phi}(X) := \inf \left\{ k > 0 : \sup_{\mu \in S} E_\mu \left[ \Phi \left( \frac{X}{k} \right) \right] \leq 1 - \alpha \right\}, \tag{5}$$

where $\alpha \in [0, 1)$ is a given parameter, $S$ is a family of capacities $\mu$ satisfying

$$\mu_0(A) = 0 \implies \mu(A) = 0 \quad \text{for} \quad A \in \mathcal{F}$$

and the function $\Phi : [0, \infty) \to [0, \infty)$ is a normalized Young function, that is strictly increasing and convex function such that $\Phi(0) = 0$ and $\Phi(1) = 1$. Moreover

$$E_\mu[X] = \int_0^\infty \mu \left( \{X > t\} \right) dt$$

is a Choquet integral with respect to the capacity $\mu$.
The aim of this talk is to characterize existance and several properties of the robust Orlicz premium defined by (5).

**Keywords:** Risk measure, Choquet integral.

## References:

[1] Bellini F., Laeven R., Gianin E. Robust return risk measures, *Mathematics and Financial Economics* 12, (2018) 5–32

[2] Chudziak, J., Rela, P., The Orlicz premium principle under uncertainty, *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas* 119, (2025) art. no. 116.

# Packings in infinite dimensions

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We will start by giving a very brief overview to some results about packings in finite dimensions. We then introduce and study the so-called *simultaneous packing and covering constant* of a Banach space, introduced by Rogers in 1950. We present new results involving this constant, focusing in particular on the extreme spaces where this constant attains its best and worst values.

**Keywords:** Packing problem, simultaneous packing and covering constant.

## References:

[1] C.A. De Bernardi, T. Russo, Ş. Sezgek, and J. Somaglia, Packings in classical Banach spaces, *arXiv*:2602.12934.

[2] C.A. De Bernardi, T. Russo, Ş. Sezgek and J. Somaglia, Tilings and coverings by balls in $\ell_1$, *arXiv*:2604.15092.

# Uniformly convex functions and their generalizations

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In this paper we present several characterizations of uniformly and locally uniformly convex functions. We also introduce certain generalizations of uniformly convex functions and investigate their basic properties.

**Keywords:** uniformly convex functions, locally uniformly convex functions, nearly uniformly convex functions, nearly uniformly smooth functions.

## References:

[1] J. M. Borwein and A. S. Lewis. Strong rotundity and optimization, *SIAM J. Optim.*, 4(1) (1994),146–158.
[2] J. M. Borwein and J. D. Vanderwerff, *Convex Functions: Constructions, Characterizations and Counterexamples*, Cambridge Univeristy Press, 2011.
[3] A. K. Chakrabarty, P. Shunmugaraj and C. Zălinescu, Continuity properties for the subdifferential and $\varepsilon$-subdifferential of a convex function and its conjugate, *J. Convex Anal.*, 14 (2007), 479–514.
[4] R. Huff, Banach spaces which are nearly uniformly convex, *Rocky Mountain J. Math.*, 10 (1980), 743–749.
[5] P. Shunmugaraj and C. Zălinescu, Compactly locally uniformly convex functions, *J. Convex Anal.*, 31(4) (2024), 1245–1272.
[6] P. Shunmugaraj and C. Zălinescu, Uniformly strongly convex Banach spaces, *Mediterr. J. Math.*, 22(6) (2025), Paper No. 141. https://doi.org/10.1007/s00009-025-02902-6.
[7] C. Zălinescu, On uniformly convex functions, *J. Math. Anal. Appl.*, 95 (1983), 344–374.
[8] C. Zălinescu, *Convex Analysis in General Vector Spaces*, World Scientific, Singapore, 2002.

# An approximation form of the Kuratowski Extension Theorem for Baire-alpha functions

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In the first part of the talk I will present a theorem in which a Baire class one function is extended by means of a uniformly convergent series of characteristic functions of certain Borel subsets of a perfectly normal space. I believe that this result can serve as a useful tool for solving a certain problem in the field of Riesz spaces, which I will discuss in the second part of the talk.

## References:

[1] Sieg, W.: An approximation form of the Kuratowski Extension Theorem for Baire-alpha functions. *Acta Mathematica Hungarica* Vol. 176, iss. 2, pp. 313-320 (2025).

# Constructing twisted sums of $C(K)$-spaces with $c_0(X)$-spaces using trees

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*Joint work with Grzegorz Plebanek.*

We introduce the notion of realization of a tree $\kappa^{\leq m}$ in $(C(K), P(K))$ and present a construction of extension of $C(K)$ with large norm of projections. We show how this method can be used to construct Banach spaces containing uncomplemented copies of $C(K)$. Finally, we present applications of this construction to some classes of Koppelberg compact spaces, including an Efimov space under CH, as well as to the space $\omega^* = \beta\omega \setminus \omega$ under the assumption of the existence of simple P-points.

**Keywords:** Functional analysis, Twisted sums, $C(K)$-spaces, Efimov space.

# Lev Bukovský, 1939–2021

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I would like to share some reflections on the life story of Professor Lev Bukovský. Most of this information is based on [1–4].

## References:

[1] Eliáš, P., Repický, M., and Šupina, J., Profesor Lev Bukovský sa dožíva významného životného jubilea, in: *Obzory matematiky, fyziky a informatiky*, volume 48, number 4, pages 56-59, 2019, ISSN 1335-4981.
[2] Lengyelfalusy, T. and Tkáčik, Š. *Lev Bukovský*, Verbum – vydavateľstvo KU, Ružomberok, Osobnosti slovenskej matematiky, volume 3, pages 62, 2021, ISBN 978-80-561-0844-4.
[3] Eliáš, P., Repický, M., and Šupina, J., Ďakujeme, niečo sme sa naučili, in: *Obzory matematiky, fyziky a informatiky*, volume 51, number 1, pages 14-20, ISSN 1335-4981, 2022.
[4] Tkáčik, Š., et all. *Košická matematická škola teórie množín a topológie: seminár prof. Leva Bukovského (1. diel).* Verbum – vydavateľstvo KU, Ružomberok, 2023, ISBN 978-80-561-1031-7.

# The invariance problem of Matkowski means under reduced regularity

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Let $I \subseteq \mathbb{R}$ be an open interval and let $f, g : I \longrightarrow \mathbb{R}$ be continuous functions which are strictly monotone in the same sense. Then

$$M_{f,g}(x, y) = (f + g)^{-1} (f(x) + g(y)) \qquad ( x, y \in I )$$

is called the *generalized weighted quasi-arithmetic mean* generated by $f$ and $g$. Since the concept was introduced by Matkowski [3], we simply refer to this class of means as *Matkowski means.* The invariance problem is the following: we are looking for those Matkowski means $M, N, K : I \times I \longrightarrow I$ which fulfill the invariance equation

$$M (N(x, y), K(x, y)) = M(x, y)$$

for all $x, y \in I$. Introducing appropriate transformations of the generators, an equivalent formulation of the invariance equation is

$$F \left(\frac{x+y}{2}\right) + f_1(x) + f_2(y) = G (g_1(x) + g_2(y)) \tag{6}$$

containing six unknown functions $F, f_1, f_2, g_1, g_2 : I \longrightarrow \mathbb{R}$ and $G : g_1(I) + g_2(I) \longrightarrow \mathbb{R}$. The general solution of equation (6) was first described by Kiss [1] assuming continuous differentiability for all of the unknown functions (along some further technical conditions). In the paper [2] the solutions have been determined supposing the mere differentiability of the functions. In this talk our purpose is to supply the characterization of the solutions assuming differentiability for only one unknown function.

The key idea is the regularity preserving property of quasisums (see [4]). More precisely, if $\varphi, \psi_1, \psi_2$ are real homeomorphisms, then the composite function $Q(x, y) = \varphi (\psi_1(x) + \psi_2(y))$ is called a *quasisum.* In [4] it is proved that if $Q$ is (continuously) differentiable, then the generators $\varphi, \psi_1, \psi_2$ have the same differentiability properties. This statement is valid for higher order differentiability as well.

**Keywords:** invariance of means, Matkowski means, quasisums, regularity of functional equations.

## References:

[1] T. Kiss, Regular solutions of a functional equation derived from the invariance problem of Matkowski means, *Aequat. Math.* 96 (2022), 1089–1124.
[2] T. Kiss, P. Tóth, Improved regularity for a composite functional equation stemming from the theory of means, *Results Math.* 81:28 (2026).
[3] J. Matkowski, Generalized weighted quasi-arithmetic means, *Aequat. Math.* 79 (2010), 203–212.
[4] P. Tóth, Regularity preservation for quasisums, *Aequat. Math.* 99 (2025), 2855–2870.

# On sets preserving upper porosity under union

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We study problem of preserving upper porosity under union by subset of the real line. Namely, for $0 < r \leq s < 1$ we consider sets that transfer $s$-porous sets to $r$-porous sets, i.e. we ask for the sets $E$ such that for every $x \in E$ and for every $F$ for which porosity of $F$ at $x$ is not less than $s$, porosity of $E \cup F$ at $x$ is not less than $r$. Such sets are called $[r, s]$-upper superporous subsets of $\mathbb{R}$.

These families generalize notions of superporosity and strong superporosity of subsets of $\mathbb{R}$ introduced in [1, 3]. Superporous sets preserve positive porosity and strongly superporous sets preserve strong porosity, i.e. if $E$ is superporous (respectively, $E$ is strongly superporous) then for every $x \in E$ and for every $F$ such that porosity of $F$ at $x$ is greater than 0 (respectively, is equal to 1), porosity of $E \cup F$ at $x$ is greater than 0 (respectively, is equal to 1).

Even though the definition and properties of $[r, s]$-superporosity, superporosity and strong superporosity are similar and all of them consist of "very small" sets, the families of these sets are essentially different. We focus on relationships between $[r, s]$-superporous sets for different indices $[r, s]$. Furthermore, we compare $[r, s]$-superporosity to superporosity and strong superporosity.

This is joint work with Stanisław Kowalczyk.

## References:

[1] V. Kelar, Topologies generated by porosity and strong porosity, *Real Anal. Exchange* 16 (1990/91), 255–267.

[2] S. Kowalczyk, M. Turowska, On $[r, s]$-Superporosity, *Symmetry* 2025, 17(1), 11; https://doi.org/10.3390/sym17010011.

[3] L. Zajíček, Porosity and $\sigma$-porosity, *Real Anal. Exchange* 13 (1987/88), 314–350.

# Adaptive Integration of Higher-Order Convex Functions

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Classical error estimates for quadrature approximations of integrals rely on derivatives of the integrand of a sufficiently high order. The required accuracy is usually achieved by establishing a sufficiently fine partition of the interval under consideration. In this talk, I propose a method for finding such a partition that is based solely on the quadrature operator being analyzed and relies exclusively on the function itself. The research focuses on functions whose derivative of a fixed odd order maintains a constant sign over the interval of integration—that is, higher-order convex (or concave) functions.

I will begin with a presentation of the now-classic result by Rowland and Varol from 1972, and then show that it can be improved in a certain sense by reducing (sometimes significantly) the number of quadrature nodes. In the survey part of the talk, I will outline the history of several years of research on adaptive methods, conducted individually as well as in collaboration with Andrzej Komisarski and Tomasz Kania.

**Keywords:** approximate integration, quadratures, Gauss quadrature, Lobatto quadrature, adaptive methods, stopping criterion, higher-order convexity.

## References:

[1] M. Bessenyei and Z. Páles. Higher-order convexity and Hermite-Hadamard-type inequalities. *Publ. Math. Debrecen*, 61(3–4):623–643, 2002.

[2] C. W. Clenshaw and A. R. Curtis. A method for numerical integration on an automatic computer. *Numer. Math.*, 2:197–205, 1960.

[3] T. Kania and S. Wąsowicz. Refined Gauss-Lobatto bounds for odd-order convexity, 2026, submitted.

[4] A. Komisarski and S. Wąsowicz. Inequalities between remainders of quadratures. *Aequationes Math.*, 91(6):1103–1114, 2017.

[5] J. H. Rowland and Y. L. Varol. Exit criteria for Simpson's compound rule. *Math. Comp.*, 26:699–703, 1972.

[6] S. Wąsowicz. On a certain adaptive method of approximate integration and its stopping criterion. *Aequationes Math.*, 94(5):887–898, 2020.

[7] S. Wąsowicz. Adaptive integration of 5-convex and 5-concave functions, 2026, submitted.

# On the continuity of functions with respect to the generalization of density topology: Solving an open problem

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We say that a sequence $\mathcal{S} = (S_n)$ of subsets of $\mathbb{R}$ is tending to zero, if $diam(S_n \cup \{0\}) \xrightarrow[n\to\infty]{} 0$. Then by $\mathbb{S}$ we denote the family of all sequences $(S_n)$ of measurable sets tending to zero such that $\lambda(S_n) > 0$, for $n \in \mathbb{N}$.

Let $\mathcal{S} = (S_n) \in \mathbb{S}$. According to the paper [1] we say that $x$ is an $\mathcal{S}$-*density point* of a measurable set $A$, if

$$\lim_{n\to\infty} \frac{\lambda((A-x)\cap S_n)}{\lambda(S_n)} = 1.$$

Moreover, for every measurable set $A$, define

$$\Phi_{\mathcal{S}}(A) := \{x \in \mathbb{R} \colon x \text{ is an } \mathcal{S}\text{-density point of } A\}.$$

Then $\Phi_{\mathcal{S}}$ is an almost lower density operator, and the family

$$\mathcal{T}_{\mathcal{S}} := \{A \in \mathcal{L} \colon A \subset \Phi_{\mathcal{S}}(A)\}$$

is a topology containing the natural topology.

Finally, let us introduce the families of continuous mappings that will be of our interest. Given $\mathcal{S}$, $\mathcal{K} \in \mathbb{S}$, define

$$\begin{aligned} \mathcal{C}_{nat,nat} &= \{f \colon (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{nat})\}, \\ \mathcal{C}_{nat,\mathcal{S}} &= \{f \colon (\mathbb{R}, \mathcal{T}_{nat}) \to (\mathbb{R}, \mathcal{T}_{\mathcal{S}})\}, \\ \mathcal{C}_{\mathcal{S},nat} &= \{f \colon (\mathbb{R}, \mathcal{T}_{\mathcal{S}}) \to (\mathbb{R}, \mathcal{T}_{nat})\}, \\ \mathcal{C}_{\mathcal{S},\mathcal{K}} &= \{f \colon (\mathbb{R}, \mathcal{T}_{\mathcal{S}}) \to (\mathbb{R}, \mathcal{T}_{\mathcal{K}})\}. \end{aligned}$$

Since $\mathcal{T}_{nat} \subset \mathcal{T}_{\mathcal{S}}$, the following inclusions hold:

$$\mathcal{C}_{nat,\mathcal{S}} \subset \mathcal{C}_{\mathcal{S},\mathcal{K}} \subset \mathcal{C}_{\mathcal{K},nat} \quad \text{and} \quad \mathcal{C}_{nat,\mathcal{S}} \subset \mathcal{C}_{nat,nat} \subset \mathcal{C}_{\mathcal{K},nat}$$

The aim of this talk is to discuss if the given inclusions are always proper.

**Keywords:** density topology, $\mathcal{S}$-density topology, continuity.

## References:

[1] F. Strobin, R. Wiertelak, On a generalization of density topologies on the real line, *Topology Appl.* 199 (2016), 1–16.

# Gateaux differentiability of locally Lipschitz convex functions defined on locally convex spaces

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The generic Gateaux differentiability of a continuous convex function on an open convex subset of a Banach space is well studied (see, e.g., R.R. Phelps' book [1]). An example of J. Rainwater [2] shows that similar results are not true when the openness of the domain is not ensured, but such properties can be obtained if the respective convex function is assumed to be locally Lipschitz, as shown by M.E. Verona [3], J. Rainwater [2], D. Noll [4] and others; of course, in such a case one says that the function is Gâteaux differentiable at a point if its subdifferential is a singleton at that point.

Our goal is to show that such results can also be obtained in the case of Gâteaux differentiability for locally Lipschitz convex functions defined on convex subsets of Hausdorff separated locally convex spaces when the domain of the function is not necessarily open. In the same framework we extend D. Gale's duality theorem [5], as well as a result of H. Ergin & T. Sarver [6] on the unique dual representation of a convex function.

**Keywords:** Lipschitz convex function; locally convex space; quasi interior; residual set.

## References:

[1] Phelps R.R.: *Convex functions, monotone operators and differentiability.* 2nd ed. (Lecture Notes in Mathematics; Vol. 1364). Springer-Verlag, Berlin; 1993.
[2] Rainwater, J.: Yet more on the differentiability of convex functions. *Proc. Amer. Math. Soc.*, 103(3) (1988), 773–778.
[3] Verona M.E.: More on the differentiability of convex functions. *Proc. Amer. Math. Soc.*, 103(1) 1988, 137–140.
[4] Noll D.: Generic Gâteaux-differentiability of convex functions on small sets. *J. Math. Anal. Appl.*, 147(2) (1990), 531–544.
[5] Gale D.: A geometric duality theorem with economic applications. *Rev. Econ. Studies*, 34(1) (1967), 19–24.
[6] Ergin H, Sarver T.: The unique minimal dual representation of a convex function. *J. Math. Anal. Appl.*, 370(2) (2010), 600–606.

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