36th International Summer Conference on Real Functions Theory

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Preface

36th International Summer Conference on Real Functions Theory has been organized by the Mathematical Institute of the Slovak Academy of Sciences following the tradition in conferences on real functions theory dating back to 1971 when it was founded by professor Tibor Šalát and professor Pavel Kostyrko.

This booklet contains the list of participants, the programme, the list of abstracts and their authors. Participants are invited to send their contributions to the journal

Tatra Mountains Mathematical Publications.

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List of Participants

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Program

Monday

9:00	9:05	**Opening**	
		chair: Małgorzata Filipczak	
9:05	9:50	M. Balcerzak	Families of equi-Baire 1 functions
9:55	10:15	J. Jachymski	A metric compactness criterion and a proof of the Ascoli–Arzelá theorem with the use of piecewise affine functions
10:20	10:40	M. Terepeta	On algebraic properties of the family of weakly Świątkowski functions
10:40	11:10	**coffee break**	
		chair: Artur Bartoszewicz	
11:10	11:30	B. Novotný	Spaces of minimal usco and minimal cusco maps as Fréchet topological vector spaces
11:35	11:55	S. Kowalczyk	On O'Malley porouscontinuous functions
12:00	12:20	M. Turowska	Compositions of ϱ-lower continuous functions

Tuesday

		chair: Marek Balcerzak	
9:00	9:45	M. Filipczak	Surprising properties of Hashimoto topologies
9:50	10:10	G. Horbaczewska	Density topologies for strictly positive Borel measures
10:15	10:35	R. Wiertelak	On properties of generalized density topologies
10:35	11:05	**coffee break**	
		chair: Małgorzata Terepeta	
11:05	11:25	A. Bartoszewicz	Variations on Olivier's Theorem
11:30	11:50	J. Llorente	Second order differentiability and related topics in the Takagi Class
11:55	12:15	P. Tóth	Strong geometric derivatives
12:20	12:40	Z. Boros	Applications of strong geometric derivatives
evening		**galla dinner**	

Wednesday

Free Day – Trip to High Tatras

Thursday

		chair: Grażyna Horbaczewska	
9:00	9:45	J. Šupina	On two applications of the ideal convergence
9:50	10:10	A. Marton	On P-like ideals induced by disjoint families
10:15	10:35	M. Doležal	Descriptive complexity of Banach spaces
10:35	11:05	**coffee break**	
		chair: Stanisław Kowalczyk	
11:05	11:25	O. Kurka	Some classes of topological spaces extending the class of Δ-spaces
11:30	11:50	F. Turoboś	Walking with Rademacher through semimetric spaces
11:55	12:15	A. Wachowicz	Baire category lower density operators with Borel values
12:20	12:40	E. Jabłońska	From the Steinhaus property to the Laczkovich one
evening		**grill**	

Friday

		chair: Zoltán Boros	
9:00	9:45	M. Papčo	Probability theory in perspective by Roman Frič
9:50	10:10	P. Eliaš	Some questions and counter-examples in measure theory motivated by categorical probability
10:15	10:35	M. Matejdes	A few variants of quasi-continuity in bitopological spaces
10:35	11:05	**coffee break**	
		chair: Milan Matejdes	
11:05	11:25	P. Nowakowski	On sequences with a unique Banach limit
11:30	11:50	E. Przemska	A unified extension of the concept of generalized closedness in topological space
11:55	12:15	S. Lindner	On the operator of center of distances

Abstracts

Families of equi-Baire 1 functions

Marek Balcerzak
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Most of the presented results are taken from the article [2] joint with O. Karlova and P. Szuca.

We study equi-Baire 1 families of functions between Polish spaces X and Y. This notion was considered by D. Lecomte [5] and A. Alighani-Koppaei [1].

We show that the respective ε-gauge in the definition of such a family can be chosen upper semi-continuous. We prove that a pointwise convergent sequence of continuous functions forms an equi-Baire 1 family. We study families of separately equi-Baire 1 functions of two variables and show that the family of all sections of separately continuous functions also forms an equi-Baire 1 family.

We characterize equi-Baire 1 families of characteristic functions. This leads us to an example witnessing that the exact counterpart of the Arzéla-Ascoli theorem for families of real-valued Baire 1 functions on $[0, 1]$ is false. On the other hand, we obtain a simple proof of the Arzéla-Ascoli type theorem for sequences of equi-Baire 1 real-valued functions in the case of pointwise convergence.

Also, we recall the game characterization of Baire 1 functions due to V. Kiss [4]. We have simplified his proof in [3]. Also, we propose a game characterizing equi-Baire 1 families of functions.

References:

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Variations on Olivier's Theorem

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The classical Olivier's theorem says that for any nonincreasing summable sequence $(a(n))$ the sequence $(na(n))$ tends to zero. This result was generalized by many authors. We propose its further generalization which implies known results. Next we consider the subset $\mathcal{AOS}$ of ℓ_1 consisting of sequences for which the assertion of Olivier's theorem is false. We study how *large* and *good* algebraic structures are contained in $\mathcal{AOS}$ and its subsets; this kind of study is known as lineability. Finally we show that $\mathcal{AOS}$ is a residual $\mathcal{G}_{\delta\sigma}$ but not an $\mathcal{F}_{\sigma\delta}$-set.

References:

[1] A. Bartoszewicz, S. Głąb, A. Widz, *Olivier's thorem: ideal convergence, algebrability and Borel classification*, RACSAM (2021)

Applications of strong geometric derivatives

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Let I denote an open interval in the real line, $0 \leq \varepsilon \in \mathbb{R}$ and $1 < p \in \mathbb{R}$. Let us consider a function $f : I \to \mathbb{R}$ that fulfills the inequality

$$f\big(\lambda x+(1-\lambda)y\big)+f\big((1-\lambda)x+\lambda y\big) \leq f(x)+f(y)+\varepsilon\,(\lambda(1-\lambda)|x-y|)^p \quad (1)$$

for every $x, y \in I$ and $\lambda \in [0,1]$. We prove that such a function f has increasing strong geometric derivatives (cf. the presentation by P. Tóth [1]). According to the representation theorem for this property [1, Theorem 2], there exist a continuously differentiable function $g : I \to \mathbb{R}$ and an additive mapping $A : \mathbb{R} \to \mathbb{R}$ such that $f(x) = g(x) + A(x)$ for every $x \in I$. This implies, in particular, that f is Wright-convex [4], i.e., it satisfies inequality (1) with $\varepsilon = 0$ as well.

Such an argument provides a new proof for Ng's decomposition theorem [2] (in the particular case when the domain is a real interval), as well as a Rolewicz type result [3] and a localization principle for Wright-convexity.

Applying a decomposition theorem for strongly geometrically differentiable functions, we perform similar investigations concerning locally approximately affine mappings.

Keywords: Wright-convex functions, approximate convexity, localization principle

References:

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Descriptive complexity of Banach spaces

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We introduce a new natural coding of separable Banach spaces. The set of codes consists of (pseudo)norms on a certain vector space and is equipped with a canonical Polish topology. We use this coding to investigate the descriptive complexities of some classical Banach spaces. Among other results, we show that ℓ_2 is

(a) the unique (up to isometry) separable Banach space with a closed isometry class,

(b) the unique (up to isomorphism) separable Banach space with an F_σ isomorphism class.

The talk is based on a joint work [1, 2] with Marek Cúth, Michal Doucha and Ondřej Kurka.

Keywords: Banach spaces, descriptive set theory

References:

[1] Cúth, Marek; Doležal, Martin; Doucha, Michal; Kurka, Ondřej: Polish spaces of Banach spaces. Forum Math. Sigma 10 (2022), Paper No. e26, 28 pp.
[2] Cúth, Marek; Doležal, Martin; Doucha, Michal; Kurka, Ondřej: Polish spaces of Banach spaces. Complexity of isometry and isomorphism classes. arXiv:2204.06834 [math.FA]

Some questions and counter-examples in measure theory motivated by categorical probability

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We discuss some natural questions occurring in the development of the foundations of probability theory viewed from the point of view of category theory. We consider the category of probability spaces and probability kernels, and analyze the non-existence of products in this category. We further analyze the notion of a g-joint observable (introduced by Roman Frič) and show how it is related to the problem of disintegration of measures on product spaces.

Surprising properties of Hashimoto topologies

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Suppose that $(X, \mathcal{T})$ is a second-countable topological space and $\mathcal{I}$ is a σ-ideal is an ideal of subsets of X, containing all singletons and such that $\mathcal{I} \cap \mathcal{T} = \varnothing$. Then the family $\mathcal{T}_{\mathcal{I}} := \{U \setminus P : U \in \mathcal{T},\ P \in \mathcal{I}\}$ is a topology called Hashimoto topology generated by $\mathcal{T}$ and $\mathcal{I}$. The notion of Hashimoto topology gives an opportunity to construct easy examples of topologies with interesting properties.

We remind some general results given by N. F. G. Martin and H. Hashimoto and focus on topologies generated by natural topology on the real line and classic shift invariant σ-ideals.

In the second part we describe families of continuous functions

$$f : (\mathbb{R}, \mathcal{T}_{\mathcal{I}}) \to (\mathbb{R}, \mathcal{T}_{\mathcal{I}})\,.$$

Density topologies for strictly positive Borel measures

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In the real analysis we often deal with the classical density topology introduced by Haupt and Pauc [13] based on the Lebesgue measure. It was deeply studied, being a source of inspirations and examples for many mathematicians [17]. It was also generalised in various ways leading also to an abstract approach described for example in [14]. We concern a very natural generalization of the basic case - density topologies defined for Borel measures. The main problem of interest is to study their separation axioms for different types of measures. We also consider homeomorphisms between such topologies. We show that there are measures generating topologies quite different than the classical density topology, for example separable and not connected. Some cases deliver us examples of spaces which are not Lindelöf but they are separable.

Keywords: separation axioms, density topologies, homeomorphism, Borel measures

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From the Steinhaus property to the Laczkovich one

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Let X be a locally compact Abelian Polish group, $\mathcal{B}(X)$ be the family of all Borel subsets of X and $\mathcal{F} \subset 2^X$. We consider the following Steinhaus' type properties:

(S^+) $\operatorname{int}(A+B) \neq \emptyset$ for every $A, B \in \mathcal{B}(X) \setminus \mathcal{F}$,

(S^-) $0 \in \operatorname{int}(A-A)$ for every $A \in \mathcal{B}(X) \setminus \mathcal{F}$,

(D^+) $A+B$ is non-meager for every $A, B \in \mathcal{B}(X) \setminus \mathcal{F}$,

(D^-) $A-A$ is non-meager in every neighborhood of 0 for every $A \in \mathcal{B}(X) \setminus \mathcal{F}$.

It is known that the family $\mathcal{M}$ of all meager sets as well as the family $\mathcal{N}$ of all sets of Haar measure zero satisfy each of these conditions. We prove that the family $\mathcal{M} \cap \mathcal{N}$ satisfies (S^-), (D^+), (D^-) although it does not satisfy (S^+) (see [3]). We also show that the σ-ideal $\sigma\overline{\mathcal{N}} \subset \mathcal{M} \cap \mathcal{N}$ generated by closed sets of Haar measure zero satisfies only (D^+) and (D^-) which leads us to the Laczkovich property [4]. This is joint work with T. Banakh, I. Banakh, Sz. Głąb and J. Swaczyna [1,2].

References:

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A metric compactness criterion and a proof of the Ascoli–Arzelá theorem with the use of piecewise affine functions

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We obtain a characterization of compact sets in metric spaces having the property that there exists an equilipschitzian family $\{F_n : n \in \mathbb{N}\}$ of completely continuous mappings such that the sequence (F_n) is pointwise convergent to the identity mapping. In particular, every Banach space having a Schauder basis possesses that property. As an aplication, we give a new proof of the classical Ascoli–Arzelá compactness theorem.

On O'Malley porouscontinuous functions

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In 2014 J. Borsík and J. Holos in [1] defined porouscontinuous functions. They defined $\mathcal{M}_r$-continuity for $r \in (0,1]$ and $\mathcal{S}_r$-continuity and $\mathcal{P}_r$-continuity for $r \in [0,1)$. They proved a sequence of proper inclusions

$$\mathcal{C}(f) \subset \mathcal{M}_1(f) \subset \mathcal{P}_s(f) \subset \mathcal{S}_s(f) \subset \mathcal{M}_s(f) \subset \\ \subset \mathcal{P}_r(f) \subset \mathcal{P}_0(f) \subset \mathcal{S}_0(f) \subset \mathcal{Q}(f)$$

($\mathcal{C}(f)$ and $\mathcal{Q}(f)$ denote the sets of all points at which $f\colon \mathbb{R} \to \mathbb{R}$ is continuous and quasicontinuous, respectively) for $0 < r < s < 1$ and $f\colon \mathbb{R} \to \mathbb{R}$. Hence,

$$\mathcal{C} \subset \mathcal{M}_1 \subset \mathcal{P}_s \subset \mathcal{S}_s \subset \mathcal{M}_s \subset \mathcal{P}_r \subset \mathcal{M}_r \subset \mathcal{P}_0 \subset \mathcal{S}_0 \subset \mathcal{Q}$$

and all inclusions are proper.

Using the notion of density in O'Malley sense (see [2,5]) we introduce new definitions of porouscontinuity, namely $\mathcal{MO}_r$-continuity for

$r \in (0,1]$ and $\mathcal{SO}_r$-continuity for $r \in [0,1)$. Then we show that for $0 < r < t < 1$ we have

$$\begin{gathered} \mathcal{C}^{\pm} = \mathcal{MO}_1 \subset \mathcal{M}_1 \subset \mathcal{P}_t \subset \mathcal{S}_t \subset \mathcal{SO}_t \subset \mathcal{MO}_t \subset \mathcal{M}_t \subset \mathcal{P}_r \subset \\ \subset \mathcal{S}_r \subset \mathcal{SO}_r \subset \mathcal{MO}_r \subset \mathcal{M}_r \subset \mathcal{P}_0 \subset \mathcal{S}_0 \subset \mathcal{SO}_0 = \mathcal{Q} \end{gathered}$$

and all inclusions are proper, ($\mathcal{C}^{\pm}$ is a family of functions right hand continuous or left hand continuous at each point).

Some relevant properties of these classes of functions are discussed.

References:

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Some classes of topological spaces extending the class of Δ-spaces

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A topological space X is said to be a Δ-*space* if for any sequence $A_1 \supset A_2 \supset \dots$ of subsets of X with $\bigcap_{n=1}^{\infty} A_n = \emptyset$, there is a sequence $G_1 \supset G_2 \supset \dots$ of open subsets of X with $A_n \subset G_n$ and $\bigcap_{n=1}^{\infty} G_n = \emptyset$. The Δ-subsets of $\mathbb{R}$ were thoroughly investigated in the past. In particular, the existence of an uncountable Δ-subset of $\mathbb{R}$ is independent of ZFC. In the setting of a general topological space, the notion of a Δ-space was first investigated by J. Kąkol and A. Leiderman.

We study several related properties of topological spaces, especially the following one. If we consider only countable sets A_n in the definition of a Δ-space, we obtain a larger class of spaces, let us call them *weakly* Δ*-spaces* for the purpose of this presentation. We show that uncountable weakly Δ-subsets of $\mathbb{R}$ exist in ZFC.

Further, a Čech-complete space X is a weakly Δ-space if and only if it is scattered (i.e., any subset of X has an isolated point). If every point of a Hausdorff space X is a G_δ-point, then X is a weakly Δ-space if and only if every countable subset of X is a G_δ-set.

The talk is based on a joint work with Jerzy Kąkol and Arkady Leiderman. For more details, see [2].

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On the operator of center of distances

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W. Bielas, S. Plewik, M. Walczyńska in the work [1] define the center of distances of the metric space (X, ρ) as

$$S(X) := \{\alpha : \forall_{x\in X} \ \exists_{y\in X} \ \rho(x,y) = \alpha\}$$

In my speech, the properties of S treated as a operator from space of compact subsets of the interval $[0, 1]$ into itself will be presented. Further examples of the applications of the operator S to determine whether a compact set is is an achievement set for some sequence will also be given.

Keywords: center of distances, achievement sets, semicontinuity

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Second order differentiability and related topics in the Takagi Class

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The Takagi function is a classical example of a continuous nowhere differentiable function, which has been studied by a large number of authors over the years. In the mid-1980s, as a generalization of the Takagi function, Hata and Yamaguti introduced a new family of functions named the Takagi class which consists of all the functions $T_w : [0,1] \to \mathbb{R}$ defined by

$$T_w(x) = \sum_{n=0}^{\infty} \frac{w_n}{2^n} \phi(2^n x)$$

where $\phi(x) = \operatorname{dist}(x, \mathbb{Z})$ and $w = (w_n)_n$ is a sequence satisfying $(2^{-n} w_n)_n \in \ell^1$. The Takagi class is a closed subspace of the space of continuous functions with the sup norm.

A few years later, Kôno carried out a deep study of the differentiability properties of the functions belonging to such class. More specifically, he proved that if $w \notin c_0$ then T_w is nowhere differentiable, if $w \in c_0 \setminus \ell^2$ then T_w is not differentiable a.e. although the range of the derivative is $\mathbb{R}$, and finally if $w \in \ell^2$ then T_w is absolutely continuous and consequently differentiable a.e.

In this talk we will show some recent results concerning the second order differentiability of the functions belonging to the Takagi class as well as the size of the sets where these properties hold. In particular, we will characterize the set of points where these functions have a Taylor expansion of order two. Moreover, we will also characterize when they satisfy a Stepanov condition of order two at a point. Finally, we will present some interesting examples.

This is a joint work with Juan Ferrera and Javier Gómez Gil. To appear in Real Analysis Exchange.

Keywords: The Takagi function, the Takagi class, Second order differentiability, convexity

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On P-like ideals induced by disjoint families

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Given two ideals $\mathcal{I}$, $\mathcal{J}$ on the set X we say that $\mathcal{I}$ is a $\mathrm{P}(\mathcal{J})$-ideal if for any countable family $\{I_n : n \in \omega\}$ of elements of $\mathcal{I}$ there is $I \in \mathcal{I}$ such that $I_n \subseteq^{\mathcal{J}} I$ (i.e. $I_n \setminus I \in \mathcal{J}$) for all $n \in \omega$. This property was introduced by M. Mačaj and M. Sleziak in [2] as a part of their study of ideal-based convergence in topological spaces and further investigated by R. Filipów and M. Staniszewski in terms of ideal convergence of real functions.

In this talk we shall present some combinatorial characterizations or descriptions of the P-like property regarding pairs of critical ideals induced by disjoint families. We mostly consider ideals determined (in some sense) by the family of functions ${}^{\omega}\omega$ and their isomorphic copies. In addition to providing some general results about this P-like property, we discuss also the importance of a particular relation – restrictive inclusion ($\mathcal{I} \subseteq^{\restriction} \mathcal{J}$) and so-called towers of monochromatic functions. If $\mathcal{I} \subseteq^{\restriction} \mathcal{J}$ characterizes "$\mathcal{I}$ is a $\mathrm{P}(\mathcal{J})$", then the discussed P-like property does not distinguish between countable and uncountable families in a sense. This is the case, for example, in the following assertion showing characterizations of P-like property between the ideal generated by the family ${}^{\omega}\omega$ and isomorphic copies of the Fubini product $\emptyset \times \mathrm{Fin}$.

Theorem. *Let $\mathcal{B}$ be an infinite partition of $\omega \times \omega$ into infinite sets. The following statements are equivalent.*

(1) $\langle {}^{\omega}\omega \rangle$ *is a* $\mathrm{P}(\langle \mathcal{B} \rangle_{\emptyset \times \mathrm{Fin}})$.

(2) $\langle {}^{\omega}\omega \rangle \subseteq^{\restriction} \langle \mathcal{B} \rangle_{\emptyset \times \mathrm{Fin}}$.

(3) *There is $k \in \omega$ such that there is no m-tower of $\mathcal{B}$-monochromatic functions (i. e. set of m infinite disjoint partial functions sharing the same domain, each being included in some $B \in \mathcal{B}$) for every $m > k$.*

(4) $(\forall \mathcal{E} \in [{}^{\omega}\omega]^{\omega})(\exists E \in [{}^{\omega}\omega]^{<\omega})(\forall f \in \mathcal{E})(\forall B \in \mathcal{B})\ |(f \cap B) \setminus \bigcup E| < \omega$.

Keywords: P-ideal, disjoint families, ideal convergence

Acknowledgement: This work was supported by the Slovak Research and Development Agency under the Contract no. APVV-20-0045.

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A few variants of quasi-continuity in bitopological spaces

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The purpose of this paper is to introduce a few variants of generalized quasi-continuity of functions defined on a bitopological space and to study their mutual relationship. Moreover, some characterization of sectional quasi-continuous function and its continuity points are investigated.

Spaces of minimal usco and minimal cusco maps as Fréchet topological vector spaces

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The notions of minimal usco and minimal cusco maps have found applications in several branches of mathematical analysis, like optimization, functional analysis, or the study of the differentiability of Lipschitz functions [1]. It is, therefore, important to know the topological properties of the spaces of these maps.

The topology of uniform convergence on compacta is one of the most important topologies on function spaces. In [2] we found conditions under which the spaces of minimal usco and minimal cusco maps equipped with the topology of uniform convergence on compacta are topological vector spaces.

We will show that spaces of these maps from a locally compact space to a Fréchet space, equipped with the topology of uniform convergence on compacta, are isomorphic as topological vector spaces. Moreover, if the domain space is also hemicompact, both spaces are Fréchet.

Keywords: minimal usco, minimal cusco, uniform convergence on compacta, topological vector space, locally convex topological vector space, isomorphism, Fréchet space.

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On sequences with a unique Banach limit

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We consider the subspaces c, $\widehat{c}$, S of ℓ^∞, where $\widehat{c}$ consists of sequences that have a unique Banach limit, and S consists of sequences whose arithmetic means of consecutive terms are convergent. We know that $c \subset \widehat{c} \subset S$. We examine the largeness of c in $\widehat{c}$, $\widehat{c}$ in S and S in ℓ^∞. We will do it from the viewpoints of porosity, algebrability and measure.

Keywords: Banach limits, porosity, algebrability, measure of families of sequences

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Probability theory in perspective by Roman Frič

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The contribution goal is to provide an overview of more then 20-years lasting research on probability theory in categorical perspective by Roman Frič.

Keywords: probability theory, category theory

A unified extension of the concept of generalized closedness in topological space

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This presentation presents a general unified approach to the notion of generalized closedness in topological spaces. The research concerning the notion of generalized closed sets in topological spaces was initiated by Levine [1] in 1970. In the succeeding years, the concepts of this type of generalizations have been investigated in many versions using the standard generalizations of topologies which has resulted in a large body of literature. However, the methods and results in the past years have become standard and lacking in innovation.

Here we introduce a general conception of a natural generalizations of families $\mathcal{B} \subset \mathcal{P}(X)$, denoted by $\mathcal{B} \triangleleft \mathcal{K}$, which are determined by other families $\mathcal{K} \subset \mathcal{P}(X)$.

We prove that the collection of all generalizations $\mathcal{B} \triangleleft \mathcal{K}$, where $\mathcal{B}, \mathcal{K} \subset \mathcal{P}(X)$, forms a Boolean algebra. The basic notion used in this conception is the closure operator designated by a family $\mathcal{B} \subset \mathcal{P}(X)$, which does not have to be the Kuratowski's operator. In this theory, the family of all generalized closed sets in a topological space (X, τ) is equal to $\mathcal{C} \triangleleft \tau$, where $\mathcal{C}$ is the family of all closed subsets of X. This concept gives tools that enable the systemizing and development of the current research area of this topic. The results obtained in this general conception, not only easily imply and generalize well-known theorems as obvious corollaries but also give many new results concerning relationships between various types of generalized closedness studied so far in a topological space. In particular, we prove and demonstrate in the graph that in a topological space (X, τ) there exist only different nine generalizations determined by the standard generalizations of topologies.

Keywords: Generalized closed set, closure operators, lattice, α-open, pre-open, γ-open, semi-open, β-open

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On two applications of the ideal convergence

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We consider the ideal version of the Fréchet–Urysohn property of a space of continuous functions. The property led to two unexpected applications. On one hand, the connection between ideal versions of the pseudointersection numbers was discovered in [2]. On the other hand, a solution to the old problem posed by J. Gerlits and Zs. Nagy [1] about distinguishing properties of the space of continuous functions was found in [3]. In our talk we shall focus on an explanation of the results, and present various related connections.

Keywords: ideal, Fréchet–Urysohn property, pseudointersection number, the space of continuous functions

Acknowledgement: This work was supported by the Slovak Research and Development Agency under the Contract no. APVV-20-0045.

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On algebraic properties of the family of weakly Świątkowski functions

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In 1978 Mańk and Świątkowski introduced the notion of now so called the Świątkowski condition [2]:

Definition 1. We say that f satisfies the Świątkowski condition (or is a Świątkowski function) if for all $x_1 \neq x_2$ with $f(x_1) < f(x_2)$ there is a point $x \in I(x_1, x_2)$ such that f is continuous at x and $f(x_1) < f(x) < f(x_2)$ ($I(x_1, x_2)$ stands for the interval with ends x_1, x_2).

Recently in the paper [1] the authors relieved this definition from demanding that f has to be continuous at x:

Definition 2. We say that f satisfies the weak Świątkowski condition (or is a weakly Świątkowski function) if for all $x_1 \neq x_2$ with $f(x_1) < f(x_2)$ there is a point $x \in I(x_1, x_2)$ such that $f(x_1) < f(x) < f(x_2)$.

In the talk we will examine some algebraic properties (among them lineability and algebrability) of families of functions related to the weakly Świątkowski condition and families of cliquish functions.

All results are obtained together with Małgorzata Filipczak and Artur Bartoszewicz.

Keywords: weak Świątkowski property, cliquish functions, algebrability, lineability, linear sensitivity

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Strong geometric derivatives

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Let I denote an open interval in the real line, and let us consider a function $f : I \to \mathbb{R}$. For $x \in I$ and $h \in \mathbb{R}$, we define the lower and upper strong geometric derivatives of f at the point x in the direction h by

$$\underline{D}_h^{\diamond} f(x) = \liminf_{\substack{y \to x \\ n \to \infty}} 2^n \left(f\left(y + \frac{h}{2^n} \right) - f(y) \right)$$

and

$$\overline{D}_h^{\diamond} f(x) = \limsup_{\substack{y \to x \\ n \to \infty}} 2^n \left(f\left(y + \frac{h}{2^n} \right) - f(y) \right),$$

respectively. We call f strongly geometrically differentiable if

$$\underline{D}_h^{\diamond} f(x) = \overline{D}_h^{\diamond} f(x) \in \mathbb{R}$$

holds for every $x \in I$ and $h \in \mathbb{R}$. We say that f has increasing strong geometric derivatives if

$$-\infty < \overline{D}_h^{\diamond} f(x) \leq \underline{D}_h^{\diamond} f(y) < +\infty$$

holds for every $h > 0$ and $x, y \in I$ such that $x < y$. These properties are characterized by the following decomposition theorems:

Theorem 1. *The function f is strongly geometrically differentiable if, and only if, there exist a continuously differentiable function $g : I \to \mathbb{R}$ and an additive mapping $A : \mathbb{R} \to \mathbb{R}$ such that $f(x) = g(x) + A(x)$ for every $x \in I$.*

Theorem 2. *The function f has increasing strong geometric derivatives if, and only if, there exist a convex function $g : I \to \mathbb{R}$ and an additive mapping $A : \mathbb{R} \to \mathbb{R}$ such that $f(x) = g(x) + A(x)$ for every $x \in I$.*

These investigations are motivated by similar concepts and related decomposition theorems, analogous to Theorem 1 [1,2].

Keywords: generalized derivatives, strong differentiability, additive mappings

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Walking with Rademacher through semimetric spaces

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A well-known result of Hans Rademacher states that if $U \subset \mathcal{R}^n$ is an open set and $f : U \to \mathcal{R}^m$ is Lipschitz, then it is differentiable almost everywhere. To extend this phenomenal result to metric (or semimetric) setting, one needs to generalize the notion of differential. For a real-valued function $f : X \to \mathcal{R}$ (where (X, d) is a metric/semimetric space), one defines the so-called *little lip* and *big lip* functions as follows:

$$\forall_{x\in X} \qquad \mathrm{lip}(f)(x) := \liminf_{r\to 0} \sup_{y\in B(x,r)} \frac{|f(y) - f(x)|}{r};$$

$$\forall_{x\in X} \qquad \mathrm{Lip}(f)(x) := \limsup_{r\to 0} \sup_{y\in B(x,r)} \frac{|f(y) - f(x)|}{r}.$$

If one replaces the real line in codomain of f with another metric/semimetric space (Y, ρ), these definitions can be refurbished in the following manner:

$$\forall_{x\in X} \qquad \mathrm{lip}(f)(x) := \liminf_{r\to 0} \sup_{y\in B(x,r)} \frac{\rho(f(y), f(x))}{r};$$

$$\forall_{x\in X} \qquad \mathrm{Lip}(f)(x) := \limsup_{r\to 0} \sup_{y\in B(x,r)} \frac{\rho(f(y), f(x))}{r}.$$

Throughout this talk we will investigate this notion of metric-metric derivation and its properties in the semimetric scope. These shall be our first steps towards the semimetric version of Rademacher theorem.

Compositions of ϱ-lower continuous functions

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In [4], compositions of approximately continuous functions were studied. Namely, properties of functions $f\colon I \to I$ such that composition $g \circ f$ is approximately continuous for every approximately continuous function g, were described. In [3], the same were done for the so-called ϱ-upper continuous functions. Properties of density preserving homeomorphisms can be found in [1].

We investigate compositions of ϱ-lower continuous functions. First, we show that continuity of $f\colon \mathbb{R} \to \mathbb{R}$ is an equivalent condition under which $f \circ g$ is ϱ-lower continuous for every ϱ-lower continuous $g\colon I \to \mathbb{R}$. Next, we consider the case of inner composition $g \circ f$, where $f\colon I \to I$ is fixed and $g\colon I \to \mathbb{R}$ is any ϱ-lower continuous function. We present some properties of functions which preserve ϱ-lower continuity under inner composition. Finally, we show properties of homeomorphisms $f\colon I \to I$ such that $g \circ f$ is ϱ-lower continuous for each ϱ-lower continuous function $g\colon I \to \mathbb{R}$.

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Baire category lower density operators with Borel values

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We prove that the lower density operator associated with the Baire category density points in the real line has Borel values of class $\mathbf{\Pi}^0_3$ which is analogous to the measure case. We also introduce the notion of the Baire category density point of a subset with the Baire property in the Cantor space, and we prove that it generates a lower density operator with Borel values of class $\mathbf{\Pi}^0_3$.

Keywords: lower density operator, density point, the Baire property, meager set, the Cantor space

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On properties of generalized density topologies

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Let $\mathcal{S}$ we will denote a **sequence** of non-degenerate and measurable sets $\{S_n\}_{n\in\mathbb{N}}$ **tending to zero**, that means $\operatorname{diam}(S_n \cup \{0\}) \underset{n\to\infty}{\longrightarrow} 0$.

In the paper [1] is presented generalization of notion of density point. We shall say that a point $x_0 \in \mathbb{R}r$ is a $\mathcal{S}$–**density point** of a set $A \in \mathcal{L}$, if

$$\lim_{n\to\infty} \frac{\lambda(A \cap (S_n + x_0))}{\lambda(S_n)} = 1.$$

Then the family

$$\mathcal{T}_S = \left\{ A \in \mathcal{L} : \underset{x\in A}{\forall}\ x \text{ is a } \mathcal{S}\text{-density point of } A \right\}$$

is a topology containing natural topology.

I am going to present properties of such defined topology.

Keywords: density topologies, $\mathcal{S}$-density topologies

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