# 34th International Summer Conference on Real Functions Theory

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# Preface

34th International Summer Conference on Real Functions Theory has been organized by the Mathematical Institute of the Slovak Academy of Sciences following the tradition in conferences on real functions theory dating back to 1971 when it was founded by professor Tibor Šalát and professor Pavel Kostyrko.

This booklet contains the list of participants, the list of abstracts and their authors. Participants are invited to send their contributions to the journal *Tatra Mountains Mathematical Publications*.

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# Abstracts

## Sandwich and Hahn-Banach theorems for invariant or equivariant vector lattice-valued operators and applications to Subdifferential Calculus and Optimization

Antonio Boccuto

Here we present some versions of Hahn-Banach, sandwich and extension theorems for vector lattice-valued operators, invariant or equivariant with respect to a fixed group $G$ of homomorphisms, given in [1] and [2]. As applications, we give some Fenchel duality and separation theorems, a version of the Moreau-Rockafellar formula and some Farkas and Kuhn-Tucker-type optimization results. Our technique is based on the existence of linear functionals, not necessarily invariant or equivariant, guaranteed by Hahn-Banach classical-like theorems, and suitable integrals for vector lattice-valued functions with respect to finitely additive real-valued measures.

**References:**

[1] Boccuto, A.: Hahn-Banach-type theorems and applications to optimization for partially ordered vector space-valued invariant operators, Real Anal. Exchange 44 (2) (2019), 333-368.

[2] Boccuto, A.: Hahn-Banach and sandwich theorems for equivariant vector lattice-valued operators and applications, Tatra Mt. Math. Publ. (2020), to appear.

# Continuity of ultrapower functions

Zoltán Boros* and Péter Tóth
University of Debrecen *presenting author

The ultrapower of an arbitrary ordered set refines its order structure providing infinitesimal feature without any algebraic structure. The construction can be extended to mappings of an ordered set into another ordered set. We compare the monotonicity and order continuity of a function with the analogous properties of its ultrapower.

# Spaces of real functions and covers

Lev Bukovský
Institute of Mathematics, P.J. Šafárik University

The paper is a survey of some recent results on relationship between covering properties of a topological space $X$ and sequence selection properties of the topological space of continuous real funtions $C(X)$ or the topological space of upper semicontinuous real funtions $USC(X)$.

Results about functions measurable in a very general way and corresponding covers, including Borel functions and Borel covers, are presented as well.

We suppose that results about covers respecting a bornology on $X$ are new.

# If I were a rich density

Rafał Filipów
University of Gdańsk

Abstract upper densities are monotone and subadditive functions from the power set of positive integers into the unit real interval that generalize the upper densities used in number theory, including the upper asymptotic density, the upper Banach density, and the upper logarithmic density.

At the open problem session of the Workshop "Densities and their application", held at St. Etienne in July 2013, G. Grekos asked a question whether there is a "nice" abstract upper density, whose the family of null sets is precisely a given ideal of subsets of $\mathbb{N}$, where "nice" would mean the properties of the familiar densities consider in number theory.

In 2018, M. Di Nasso and R. Jin (Acta Arith. 185 (2018), no. 4) showed that the answer is positive for the summable ideals (for instance, the family of finite sets and the family of sequences whose series of reciprocals converge) when "nice" density means translation invariant and rich density (i.e. density which is onto the unit interval).

In my talk I show how to extend their result to all ideals with the Baire property. This extension was obtained jointly with Jacek Tryba and the results are published in the paper "Densities for sets of natural numbers vanishing on a given family" in J. of Number Theory 211 (2020), 371-382.

# Łukasiewicz logic and the divisible extension of probability theory

Roman Frič

Mathematical Institute, Slovak Academy of Sciences

We show that measurable fuzzy sets carrying the multivalued Łukasiewicz logic lead to a natural generalization of the classical Kolmogorovian probability theory. The transition from Boolean logic to Łukasiewicz logic has a categorical background and the resulting divisible probability theory possesses both fuzzy and quantum qualities. In the divisible probability theory, morphisms are called observables and play analogous role as classical random variables - convey stochastic information from one system of random events to another one. Observables preserving the Łukasiewicz logic are called conservative. They are exactly observables that characterize the "classical core" of divisible probability theory. They send crisp random events to crisp random events and Dirac probability measures to Dirac probability measures. The nonclassical observables send some crisp random events to genuine fuzzy events and some Dirac probability measures to nondegenerated probability measures. They constitute the added value of transition from classical to divisible probability theory.

At previous RF conferences we have presented our results related to the transition from classical probability space $(\Omega, \mathbf{A}, p)$ to its fuzzification $(\Omega, \mathcal{M}(\mathbf{A}), \int(\cdot)\mathrm{d}p)$, where $\mathcal{M}(\mathbf{A})$ is the set of all measurable functions into $[0,1]$ and $\int(\cdot)\mathrm{d}p$ is the probability integral with respect to $p$.

In the present contribution we clarify the role of Łukasiewicz logic in the transition. Real functions, measure and integration play a vital role.

# Quasicontinuous functions and cardinal invariants of the topology of pointwise convergence

Ľubica Holá[1*] and Dušan Holý[2]
[1]Mathematical Institute, Slovak Academy of Sciences *presenting author
[2]Trnava University

Let $X$ be a Hausdorff topological space and let $Q(X, \mathbb{R})$ be the space of all quasicontinuous functions on $X$ with values in $\mathbb{R}$ and $\tau_p$ be the topology of pointwise convergence. We prove that $Q(X, \mathbb{R})$ is dense in $\mathbb{R}^X$ equipped with the product topology. We characterize some cardinal invariants of $(Q(X, \mathbb{R}), \tau_p)$. We also compare cardinal invariants of $(Q(\mathbb{R}, \mathbb{R}), \tau_p)$ and $(C(\mathbb{R}, \mathbb{R}), \tau_p)$, the space of all continuous functions on $\mathbb{R}$ with values in $\mathbb{R}$.

# Quasicontinuous functions and cardinal invariants of the topology of uniform convergence on compacta

Ľubica Holá[1] and Dušan Holý[2*]

[1]Mathematical Institute, Slovak Academy of Sciences
[2]Trnava University *presenting author

Let $X$ be a Hausdorff topological space, $Q(X, \mathbb{R})$ be the space of all quasicontinuous functions on $X$ with values in $\mathbb{R}$ and $\tau_{UC}$ be the topology of uniform convergence on compacta. If $X$ is hemicompact, then $(Q(X, \mathbb{R}), \tau_{UC})$ is metrizable and thus many cardinal invariants, including weight, density and cellularity coincide on $(Q(X, \mathbb{R}), \tau_{UC})$. We find further conditions on $X$ under which these cardinal invariants coincide on $(Q(X, \mathbb{R}), \tau_{UC})$ as well as characterizations of some cardinal invariants of $(Q(X, \mathbb{R}), \tau_{UC})$. It is known that the weight of continuous functions $(C(\mathbb{R}, \mathbb{R}), \tau_{UC})$ is $\aleph_0$. We will show that the weight of $(Q(\mathbb{R}, \mathbb{R}), \tau_{UC})$ is $2^{\mathfrak{c}}$.

# Hindman, Folkman and van der Waerden space

Krzysztof Kowitz
University of Gdańsk

We recall the definitions of the ideals of Hindman, Folkman, and van der Waerden. We show examples of sets that belong to these ideals or not. We define the spaces associated with these ideals, we recall a few known facts about the relations between these spaces. Regarding this, we introduce the theorems under the assumption of the continuum hypothesis (A. Kwela) and Martin's axiom. We end the presentation with questions about the unknown relations between these spaces.

# Rizza's ideal and a comparison of some known set-theoretical ideals from number theory and combinatorics

Marta Kwela
University of Gdańsk

We introduce a new ideal, related to the topology of Rizza defined on the set of positive integers, and examine its basic properties. Moreover, we recall the definitions of some well-known set-theoretic ideals that appear, e.g., in number theory and combinatorics, and present a comparison of all mentioned ideals – we explore all of the possible inclusions between them.

# On b-metric preserving functions

Mateusz Lichman,* Piotr Nowakowski, and Filip Turoboś
Łódź University of Technology *presenting author

In this presentation, we focus on functions that enable us to introduce a b-metric on a finite product of b-metric spaces. These functions we will call b-metric preserving. First, we propose sufficient conditions simultaneously showing that these are not necessary. Later, we consider functions that merge a collection of metrics into a single b-metric and their relation to b-metric preserving functions. Finally, we establish a complete characterisation of both mentioned classes of functions.

# When are topological vector spaces of minimal usco and minimal cusco maps isomorphic?

Branislav Novotný* and Ľubica Holá

Mathematical Institute, Slovak Academy of Sciences *presenting author

A multifunction is usco (cusco), if it is upper semicontinuous with non-empty compact (and convex) values. It is minimal usco/cusco if it is usco/cusco and minimal with respect to graph inclusion. These set-valued maps have various applications and therefore it is interesting to study their properties. They describe common features of maximal monotone operators, of the convex subdifferential and of Clarke generalized gradient. Minimal cusco maps appear in the study of weak Asplund spaces [2],[5], optimization [6] and in the study of differentiability of Lipschitz functions [1], [4].

We consider the spaces of minimal usco (resp. cusco) maps from $X$ to $Y$. We showed that if $X$ is a Baire space and $Y$ is a Stegall locally convex space, we can define a vector structure on these spaces. For some considered topologies, they are topological vector spaces, see [3]. We are interested when are these topological vector spaces of minimal usco and minimal cusco maps isomorphic.

**References:**

[1] Borwein, J. M.: Fixed point theory and applications, vol. 252 of Pitman Res. Notes Math. Ser. Longman, Harlow, 1991, chap. Minimal cuscos and subgradients of Lipschitz functions, pp. 57–81.

[2] Fabian, M. J.: Gâteaux differentiability of convex functions and topology: weak Asplund spaces. Canad. Math. Soc. Ser. Monogr. Adv. Texts. Wiley, New York, 1997.

[3] Holá, Ľ. and Novotný, B.: When is the space of minimal usco/cusco maps a topological vector space. Journal of Mathematical Analysis and Applications 489, no. 1 (sep 2020), 124125.

[4] Moors, W.: A characterization of minimal subdifferential mappings of locally Lipschitz functions. Set-Valued Analysis 3 (1995), 129–141.

[5] Moors, W. and Somasundaram, S. A: Gateaux differentiability space that is not weak Asplund. Proc. Amer. Math. Soc. 134 (2006), 2745–2754.

[6] Čoban, M. M., Kenderov, P. S., and Revalski, J. P.: Generic well-posedness of optimization problems in topological spaces. Mathematika 36 (1989), 310–324.

# Packing dimension of a central Cantor set

Piotr Nowakowski

Łódź University of Technology

We consider the family $\mathcal{CS}$ of central Cantor subsets of $[0, 1]$. Each set in $\mathcal{CS}$ is uniquely determined by a sequence $a = (a_n)$ belonging to the Polish space $X := (0, 1)^{\mathbb{N}}$ equipped with probabilility product measure $\mu$. This yields a one-to-one correspondence between sets in $\mathcal{CS}$ and sequences in $X$. If $\mathcal{A} \subset \mathcal{CS}$, the corresponding subset of $X$ is denoted by $\mathcal{A}^*$. In [Balcerzak, Filipczak, N., 2019] there were studied the families of sets with Haudsdorff dimension 0, strongly porous sets, porous sets, microscopic sets and null sets in $\mathcal{CS}$. Some of above families occured to be of $\mu$ measure zero and some of full measure. Besides, all of above families are residual, while we treat them as the corresponding families in $X$. In this presentation we want to examine one more family of small central Cantor sets, that is, the family of sets with packing dimension equal to zero. We will establish its measure and Baire category.

## On a relation between thin sets and xor-sets

Paweł Pasteczka

We study properties of thin sets introduced recently by T. Banakh and E. Jabłońska. Using Banach-Mazur games we prove that all thin sets are Baire spaces and generic (in particular maximal) thin sets are not Borel.

We also show that the Cantor cube can be decomposed to two sets of this type. This result is related to so-called xor-sets defined by D. Niwiński and E. Kopczyński in 2014.

## Regular sets in topological spaces

Emilia Przemska

The question as to the number of sets obtainable from a given subset of a topological space using the operators derived by composing members of the set $\{b, i, \vee, \wedge\}$, where $b$, $i$, $\vee$ and $\wedge$ denote the closure operator, the interior operator, the binary operators corresponding to union and intersection, respectively, is called the Kuratowski $\{b, i, \vee, \wedge\}$ - problem. This problem has been solved independently by Sherman [2] and, Gardner and Jackson [1], where the resulting 34 plus identity operators were depicted in the Hasse diagram. In this paper we investigate the sets of fixed points of these operators. We show that there are at most 23 such families of subsets. Twelve of them are the topology, the family of all closed subsets plus, well known generalizations of open sets, plus the families of their complements. Each of the other 11 families forms a complete complemented lattice under the operations of join, meet and negation defined according to a uniform procedure. Two of them are the well known Boolean algebras formed by the regular open sets and regular closed sets, any of the others in general need not be a Boolean algebras.

**References:**

[1] Gardner, B. J,. and Jackson, M. G.: The Kuratowski closure-complement theorem, New Zealand J. Math., 38 (2008), 9–44.

[2] Sherman, D.: Variations on Kuratowski's 14-set theorem, The American Mathematical Monthly, 117 (2) (2010), 113–123.

# An important property of topologically semi-open and preopen sets

Muwafaq Mahdi Salih
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In particular, a family $\mathcal{R}$ of relations on a set $X$ is called a relator on $X$, and the ordered pair $X(\mathcal{R}) = (X, \mathcal{R})$ is called a relator space. Relator spaces of this simpler type are already substantial generalizations of not only ordered sets and uniform spaces, but also topological, closure and proximity spaces.

As it is usual, for any $x \in X$ and $A \subseteq X$, we write

(1) $x \in \operatorname{int}_{\mathcal{R}}(A)$ if $R(x) \subseteq A$ for some $R \in \mathcal{R}$;

(2) $x \in \operatorname{cl}_{\mathcal{R}}(A)$ if $R(x) \cap A \neq \emptyset$ for all $R \in \mathcal{R}$;

(3) $A \in \mathcal{T}_{\mathcal{R}}^{s}$ if $A \subseteq \operatorname{cl}_{\mathcal{R}}\big(\operatorname{int}_{\mathcal{R}}(A)\big)$; (4) $A \in \mathcal{T}_{\mathcal{R}}^{p}$ if $A \subseteq \operatorname{int}_{\mathcal{R}}\big(\operatorname{cl}_{\mathcal{R}}(A)\big)$.

Extending and improving some of the statements of Takashi Noiri (1984) and Julian Dontchev (1998), we prove that if $\mathcal{R}$ is a topologically filtered, topological relator on $X$, and

$$A \in \mathcal{T}_{\mathcal{R}}^{s}, \qquad B \in \mathcal{T}_{\mathcal{R}}^{p}, \qquad \operatorname{int}_{\mathcal{R}}(A) \subseteq C \subseteq \operatorname{cl}_{\mathcal{R}}(A), \qquad B \subseteq D \subseteq X,$$

then $\operatorname{cl}_{\mathcal{R}}(A) \cap B = \operatorname{cl}_{\mathcal{R}}(C \cap D) \cap B$.

Hence, by choosing $C$ and $D$ appropriately, we can immediately derive a great number equalities for the set $\operatorname{cl}_{\mathcal{R}}(A) \cap B$. For instance, we can at once state that

$$\operatorname{cl}_{\mathcal{R}}(A) \cap B = \operatorname{cl}_{\mathcal{R}}\Big(\operatorname{cl}_{\mathcal{R}}\big(\operatorname{int}_{\mathcal{R}}(A)\big) \cap \operatorname{int}_{\mathcal{R}}\big(\operatorname{cl}_{\mathcal{R}}(B)\big)\Big) \cap B\,.$$

The above results, together with a simple counterexample to a statement of Julian Dontchev, are under publication in a very long joint paper with Themistocles M. Rassias and Árpád Száz.

# On some properties of Kurzweil–Henstock type integral on zero-dimensional compact group

Valentin Skvortsov

Moscow, Russia

Here we study whether some results on differentiation of generalized Kurzweil–Henstock type integrals known in the scalar-valued case remain valid for Banach-space-valued case and consider application to some problems in harmonic analysis on compact abelian zero-dimensional groups which depend on or can be reduced to the corresponding problems in the theory of differentiation or integration of Banach-valued functions on such a group.

We define a derivation basis on a group $G$ with the above mentioned properties, introduce a notion of derivative (strong and weak) of a Banach-valued function with respect to this basis and show that a Henstock-Bochner type integral (or Henstock-Pettis type in the weak case) solves the problem of recovering a differentiable function from its respective derivative for any Banach space. In the opposite direction, the statement on differentiation of the indefinite Henstock type integral is true only for Banach spaces of finite dimension. Namely we prove that for any infinite-dimensional Banach space there exists a function on a group $G$, with values in this space, integrable on $G$ in the sense of the Henstock-Bochner-type integral with the indefinite integral being nowhere differentiable with respect to the derivation basis introduced on the group.

A relation between convergence properties of a series with respect to the system of characters of the group $G$ and differential properties of a certain function associated with the series is established and in this way the problem of recovering, by generalized Fourier formulae, the Banach-space-valued coefficients of a convergent series is solved for any Banach space by reducing it to the above mentioned problem of recovering a function from its derivative using the considered Henstock-type integral.

Some results related to the convergence of Fourier-Henstock series with respect to characters are obtained. In particular it follows that for any infinite-dimensional Banach space there exists a function with values in this space such that its Fourier-Henstock series diverges everywhere on $G$. It is also shown that the rate of growth of the partial sums of such a divergent series depends on a structure of the considered Banach space.

Some of the presented here results are a generalization of those ones obtained in [1] and [2]. The details are in the attached preprint [3].

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**References:**

[1] Dilworth, S. J., Girardi M.: Nowhere weak differentiability of the Pettis integral, Quaest. Math. 18 (1995) no. 4, 365–380.

[2] Skvortsov, V. A.: Integration of Banach-valued functions and Haar series with Banach-valued coefficients, Moscow Univ. Math. Bull. 72 (2017) no. 1, 24–30.

[3] Skvortsov, V. A.: Recovering Banach-valued coefficients of series with respect to characters of zero-dimensional groups, to be published in Annales Univ. Sci. Budapest 2020.

# Super and hyper products of super relations

Árpád Száz
University of Debrecen

If $R$ is a relation on $X$ to $Y$, $U$ is a relation on $\mathcal{P}(X)$ to $Y$, and $V$ is a relation on $\mathcal{P}(X)$ to $\mathcal{P}(Y)$, then we say that $R$ is an ordinary relation, $U$ is a super relation, and $V$ is a hyper relation on $X$ to $Y$.

By using an ingenious idea of Emilia Przemska, on a unified treatment of generalized open sets, we shall consider here some reasonable notions of product relations for super, and thus in particular also for ordinary relations.

# A non-conventional three relator space whose very particular cases can be used to treat the various generalized open sets in a unified way

Themistocles M. Rassias[1] and Árpád Száz[2*]

[1]National Technical University of Athens
[2]University of Debrecen *presenting author

If $\mathcal{R}$ is a family of relations on $X$ to $Y$, $\mathcal{U}$ is a family of relations on $\mathcal{P}(X)$ to $Y$, and $\mathcal{V}$ is a family of relations on $\mathcal{P}(X)$ to $\mathcal{P}(Y)$, then we say that $\mathcal{R}$ is an ordinary relator, $\mathcal{U}$ is a super relator, and $\mathcal{V}$ is a hyper relator on $X$ to $Y$.

We show that particular cases of the $X = Y$ particular case of the almost simple three relator space $(X, Y)(\mathcal{R}, U, V)$, where $U \in \mathcal{U}$ and $V \in \mathcal{V}$, can be used to treat, in a unified way, the various generalized open sets having been studied by a surprisingly great number of topologists.

# Quasicontinuous functions and cardinal invariants of the topology of pointwise convergence

Małgorzata Filipczak[1] and Małgorzata Terepeta[2*]
[1]University of Łódź
[2]Łódź University of Technology *presenting author

We examine some topologies of density-type (called $(\psi, n)$-density topologies) obtained as a result of strengthening the Lebesgue Density Theorem.

It turnes out that these topologies are the generalizations of superdensity, enhanced density and $m$-density topologies and have some applications in the theory of sets of finite perimeter and in Sobolev spaces.

[1] S. Delladio, Functions of class $C^1$ subject to a Legendre condition in an enhanced density set, Rev. Mat. Iberoam. 28.1 (2012), 127–140.

[2] S. Delladio, A note on some topological properties of sets with finite perimeter, Glasgow Math. J. 58(3) (2015), 637–647.

[3] S. Delladio, Approximate continuity and differentiability with respect to density degree. An application to BV and Sobolev functions, Preprint.

[4] M. Filipczak, M. Terepeta, On $\psi$-density topologies on the real line and on the plane, Traditional and present-day topics in real analysis, Faculty of Mathematics and Computer Science, University of Łódź, 2013, 367–387.

# Finite semimetric spaces and relaxed polygonal inequality

Filip Turoboś

Along with the arrival of new century, the problem with dealing with large amounts of data has appeared. The rise of the concept of big data proved that the technology itself is not enough to provide us with reliable, large-scale databases and exerted pressure on the researchers to come up with new algorithms which would enable more efficient browsing through large collections of information. This, in turn, takes us to the world of metric spaces, which turned to be invaluable asset in nowadays data analysis.

However, introducing a metric in the set of points which are described by multitude of different values might be a tedious task – thus the need for new mathematical tools arises. Several approaches to the problem were taken. Some researchers started their work on the functions combining multiple metrics (defined for each type of value stored in the record) into a single metric defined on a product space. In this presentation we will focus on the other approach, i.e. we take a closer look on the weaker versions of the metric space axioms and we will present a simple and computationally quick method of determining the relaxing constant in the condition known in the literature as c-relaxed polygonal inequality.

In the second part of the presentation, we turn our attention to the class of spaces known as doubling metric spaces (or rather – their natural generalizations). We consider ambiguity of notion of a bounded set in these generalized spaces. Additionally, we prove that the Assouad embedding theorem can be extended to the scope of b-metric spaces but not further.

# On continuity in generalized topology

Małgorzata Turowska* and Stanisław Kowalczyk
Pomeranian Academy in Słupsk *presenting author

In many fields of mathematics, especially in the theory of real functions, different kinds of generalized continuity are considered. The aim of our presentation is unification of properties of sets of points of these generalized continuities. The useful tool for this purpose is the notion of a generalized topology introduced by Á. Császár in 2002 in [3]. From this time properties of generalized topological spaces are intensively studied, see for example [1,2,4]. It turned out that many of previously considered types of generalized continuities may be equivalently defined as a continuity in a generalized topology.

We characterize sets of points of continuity for functions defined on a generalized topological space. We introduce definition of a topology associated with given generalized topology. It turns out there is a strict relationship between continuity in a generalized topology and continuity in an associated topology.

**References:**

[1] C. Cao, J. Yan, W. Wang, B. Wang, Some generalized continuities functions on generalized topological spaces, Hacet. J. Math. Stat. 42(2) (2013), 159–163.

[2] Á. Császár, Closures of open sets in generalized topological spaces, Ann. Univ. Sci. Budapest. Eötvös Sect. Math. 47 (2004), 123–126.

[3] Á. Császár, Generalized topology, generalized continuity, Acta Math. Hungar. 96 (2002), no. 4, 351–357.

[4] Á. Császár, Product of generalized topologies, Acta Math. Hungar. 123 (1-2) (2009), 127–132.

## Nested intervals property for ultrapowers of ordered sets

Zoltán Boros and Péter Tóth*
University of Debrecen *presenting author

The ultrapower $\mathbb{Q}^*$ of the field of rational numbers was constructed by Stroyan and Luxemburg in order to give a representation of infinitesimal numbers as elements of the ordered field $\mathbb{Q}^*$. In a recent article, Corazza proved the nested intervals property of $\mathbb{Q}^*$. In our presentation we introduce the ultrapower $T^*$ of an arbitrary ordered set $T$, and show that an ultrapower typically fulfills the nested intervals properties - for open and closed intervals as well. We also claim that the ultrapower of an ordered set usually does not satisfy the axiom of completeness.

# An extension of the Abel-Liouville identity

Zsolt Páles[1] and Amr Zakaria[2*]
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In this note, we present an extension of the celebrated Abel–Liouville identity by introducing the notion of generalized Wronskians in terms of noncommutative complete Bell polynomials. We also characterize a certain equivalence of n-dimensional vector-valued functions in the subclass of n-times differentiable functions with a nonvanishing Wronskian.

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