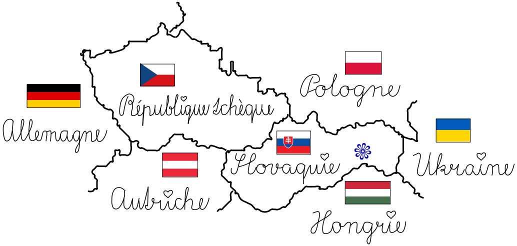
On the Descriptive Complexity of $\overline{\Sigma^* \overline{L}}$

Galina Jirásková

Mathematical Institute, Slovak Academy of Sciences, Košice, Slovakia



Joint work with Michal Hospodár and Peter Mlynárčik

DLT 2017, Liège, Belgium

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Finite automata: deterministic, nondeterministic, alternating

$$A = (Q, \Sigma, \delta, s, F)$$

- Q is a non-empty finite set of states
- Σ is an input alphabet
- $s \in Q$ is the starting state
- $F \subseteq Q$ is the set of final states
- δ is the transition function from $Q \times \Sigma$ to
  - a single state in DFA
  - a union of states in NFA
  - a boolean function of states in AFA

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Example

DFA: δ		a	b
$\to$	q_1	q_2	q_2
$\circledcirc$	q_2	q_2	q_1

NFA: δ		a	b
$\to$	q_1	q_2	$q_1 \vee q_2$
$\circledcirc$	q_2	q_1	q_1

AFA: δ		a	b
$\to$	q_1	q_2	$q_1 \wedge \overline{q_2}$
$\circledcirc$	q_2	q_1	$\overline{q_1}$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Example

DFA: δ		a	b
$\to$	q_1	q_2	q_2
$\circledcirc$	q_2	q_2	q_1

$\delta(q_1, ba) =$
$\delta(q_2, a) = q_2$

- final state

NFA: δ		a	b
$\to$	q_1	q_2	$q_1 \vee q_2$
$\circledcirc$	q_2	q_1	q_1

$\delta(q_1, ba) =$
$\delta(q_1 \vee q_2, a) = q_2 \vee q_1$

- contains final state

AFA: δ		a	b
$\to$	q_1	q_2	$q_1 \wedge \overline{q_2}$
$\circledcirc$	q_2	q_1	$\overline{q_1}$

$\delta(q_1, ba) =$
$\delta(q_1 \wedge \overline{q_2}, a) = q_2 \wedge \overline{q_1}$

- evaluate at finality vector $f = (0, 1)$
- gives 1; accepts ba

Outline

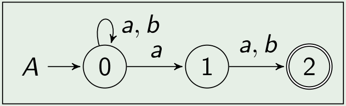
1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

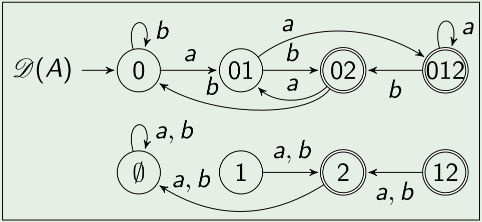
Subset automaton of an NFA $A = (Q, \Sigma, \delta, s, F)$

is the DFA

$$\mathscr{D}(A) = (2^Q, \Sigma, \delta, \{s\}, \{S \in 2^Q \mid S \cap F \neq \emptyset\})$$

Example ("ab-automaton"; $n = 3$)





Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

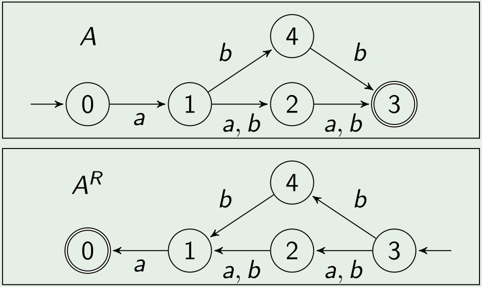
The reverse of an NFA $A = (Q, \Sigma, \delta, s, F)$

is the NFA

$$A^R = (Q, \Sigma, \delta^R, F, \{s\})$$

where $(p, a, q) \in \delta^R$ iff $(q, a, p) \in \delta$

Example (Reverse of NFA)



Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

A general formulation of the problem:

Dokl. Akad. Nauk SSSR
Tom 194 (1970), No. 6

Soviet Math. Dokl.
Vol. 11 (1970), No. 5

ESTIMATES OF THE NUMBER OF STATES OF FINITE AUTOMATA

519.95

A. N. MASLOV

A general formulation of the problem is as follows: We have events $T(A_i)$ $(1 \leq i \leq k)$ representable in automata A_i with n_i states, respectively, and a k-place operation f on events, preserving representability in finite automata. What is the maximal number of states of a minimal automaton representing $f(T(A_1), \cdots, T(A_k))$, for the given n_i?

We have languages L_i $(1 \leq i \leq k)$ represented by automata A_i with n_i states, resp., and a k-ary regular operation f.

What is the maximal number of states of a minimal automaton representing $f(L_1, \ldots, L_k)$, for the given n_i?

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \rightarrow b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

A general formulation of the problem:

We have languages L_i $(1 \leq i \leq k)$ represented by automata A_i with n_i states, resp., and a k-ary regular operation f.

What is the maximal number of states of a minimal automaton representing $f(L_1, \ldots, L_k)$, for the given n_i?

In this paper:

- $k = 1$
- $f(L) = \overline{\Sigma^* \overline{L}}$
- automaton for L in {DFA,NFA,AFA}
- automaton for $\overline{\Sigma^* \overline{L}}$ in {DFA,NFA,AFA}

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \rightarrow b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Motivation and history I

- J. C. Birget (1996): The state complexity of $\overline{\Sigma^* \overline{L}}$ and its connection with temporal logic
- combined operation with complementation
  - operational state complexity (Maslov 1970, Yu, Zhuang, Salomaa 1994)
  - combined operations (A. Salomaa, K. Salomaa, Yu 2007)

    ...
  - star-complement-star (Jiraskova, Shallit 2012: $2^{\Theta(n \log n)}$)
  - boundary, i.e. $L^* \cap (L^c)^*$ (Jiraskova, Jirasek 2013: $\Theta(4^n)$)

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Question by Jean-Éric Pin

- If NFA (DFA) for L has n states, how many states has NFA (DFA) for $\overline{\Sigma^* \overline{L}}$?

Restricted Temporal Logic (Cohen, Perrin, Pin)

- models of φ = a regular language $L(\varphi)$
- temporal operators:
  $\circ$ = "next"
  $\diamond$ = "eventually"
- combined operator: $\Box$ ("forever")
  $\Box = -\diamond-$ ("not eventually not")

$$L(\overline{\varphi}) = \overline{L(\varphi)} \qquad L(\diamond\varphi) = \Sigma^* L(\varphi) \quad \Rightarrow \quad L(\Box\varphi) = \overline{L(\diamond\overline{\varphi})} = \overline{\Sigma^* \overline{L(\varphi)}}$$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Properties of the forever operator

- $b(L) \subseteq L$
- $b(L) = \{w \in L \mid \text{every suffix of } w \text{ is in } L\}$
- if L is suffix-closed, then $b(L) = L$
- if $\varepsilon \notin L$, then $b(L) = \emptyset$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

- Upper bounds:
  - L accepted by an n-state DFA

  $\Rightarrow b(L)$ accepted by a 2^{n-1}-state DFA
  - L accepted by an n-state NFA

  $\Rightarrow b(L)$ accepted by a $2^{n+1}+1$-state NFA
- Lower bounds:
  - there is L acc. by ternary n-state DFA s.t. every NFA for $b(L)$ has at least 2^{n-1} states

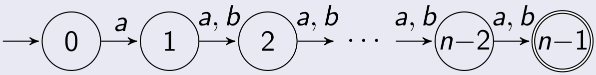
$L \backslash b(L)$	DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1}+1$	3	
AFA					$\leq n+1$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

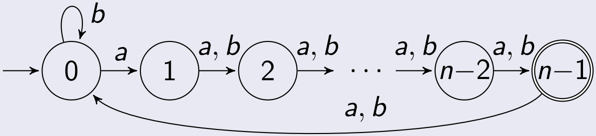
DFA-to-DFA; lower bound$= 2^{n-1}$ with $|\Sigma| = 2$

- Birget's claim: $L^c = a(a+b)^{n-2}$



However, complete DFA has $n+1$ states.

- Our modification: $L^c =$



- Unary case: n

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1} + 1$	3	
AFA					$\leq n + 1$

Our improvements and new results

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	AFA
DFA	2^{n-1}	2			
NFA					
AFA					

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Reverse of the forever operator:

$$b(L)^R = \overline{\overline{L^R}\Sigma^*}$$

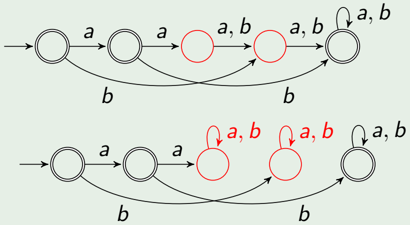
- L^R is accepted by DFA A

$\Rightarrow$ $b(L)^R$ is accepted by DFA obtained from A by replacing every non-final state with the non-final sink state

Example

L^R

$b(L)^R$



Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Leiss 1981, FJY 1990

L has n-state AFA
if and only if
L^R has 2^n-state DFA with 2^{n-1} final states

Reverse of the forever operator:

$$b(L)^R = \overline{\overline{L^R}\Sigma^*}$$

- L^R is accepted by DFA A

$\Rightarrow$ $b(L)^R$ is accepted by DFA obtained from A by replacing every non-final state with the non-final sink state

AFA-to-AFA: upper bound $= n$

- L has n-state AFA

$\Rightarrow$ L^R has 2^n-state DFA with 2^{n-1} final states
$\Rightarrow$ $b(L)^R$ has 2^n-state DFA with 2^{n-1} final st.
$\Rightarrow$ $b(L)$ has n-state AFA

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

$L \backslash b(L)$	DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1}+1$	3	
AFA					$\leq n+1$

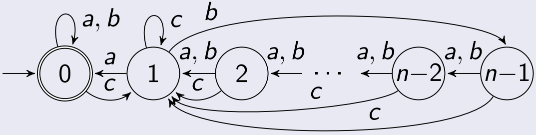
Our improvements and new results

$L \backslash b(L)$	DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	AFA
DFA	2^{n-1}	2			$\leq n$
NFA			$\leq 2^n+1$		$\leq n$
AFA					$\leq n$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

DFA-to-AFA: lower bound $= n$ with $|\Sigma| = 3$



- L is suffix-closed $\Rightarrow$ $b(L) = L$
- minimal DFA for L^R has $2^{n-1} + 1$ states
  $\Rightarrow$ every AFA for L has $\geq n$ states

Leiss 1981, FJY 1990

L has n-state AFA

if and only if

L^R has 2^n-state DFA with 2^{n-1} final states

AFA-to-AFA: lower bound $= n$ with $|\Sigma| = 1$

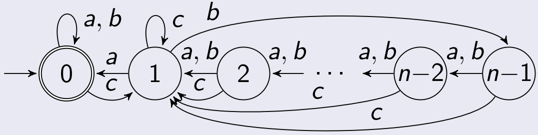
$$L = \{a^i \mid 0 \leq i \leq 2^{n-1} - 1\}$$

- accepted by DFA of 2^n states; 2^{n-1} final
  $\Rightarrow$ accepted by n-state AFA; . . .

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

DFA-to-AFA: lower bound $= n$ with $|\Sigma| = 3$

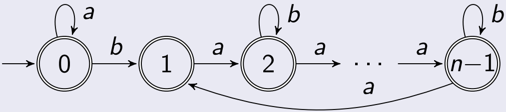


- L is suffix-closed $\Rightarrow$ $b(L) = L$
- minimal DFA for L^R has $2^{n-1} + 1$ states $\Rightarrow$ every AFA for L has $\geq n$ states

Leiss 1981, FJY 1990

L has n-state AFA
if and only if
L^R has 2^n-state DFA with 2^{n-1} final states

NFA-to-AFA: lower bound $= n$ with $|\Sigma| = 2$



Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1}+1$	3	
AFA					$\leq n+1$

Our improvements and new results

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	AFA	$\|\Sigma\|$
DFA	2^{n-1}	2		n	3
NFA			$\leq 2^n+1$	n	2
AFA				n	1

$$b(L)^R = \overline{\overline{L^R}\Sigma^*}$$

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

NFA-to-NFA: upper bound $= 2^{n-1}$

•	L	n-state NFA (one initial)
$\Rightarrow$	L^R	n-state NFA (one final)
$\Rightarrow$	L^R	2^n -state DFA (2^{n-1} final)
$\Rightarrow$	$\overline{L^R}$	2^n -state DFA (2^{n-1} final)
$\Rightarrow$	$\overline{L^R}\Sigma^*$	$2^{n-1}+1$ -state DFA (one final sink)
$\Rightarrow$	$\overline{\overline{L^R}\Sigma^*}$	2^{n-1} state partial DFA D (all final)
$\Rightarrow$	$b(L)$	$2^{n-1}+1$ -state NFA

- show that $L(D)$ is accepted by a 2^{n-1} -state NFA with one final state

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^*\overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

$L \backslash b(L)$	DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1}+1$	3	
AFA					$\leq n+1$

Our improvements and new results

$L \backslash b(L)$	DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	AFA	$\lvert\Sigma\rvert$
DFA	2^{n-1}	2			n	3
NFA			2^{n-1}	3	n	2
AFA					n	1

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Dedekind number $M(n) =$

- the number of antichains of subsets of an n-element set

Example

n	$M(n)$
0	2
1	3
2	6
3	20
4	168
5	7 581
6	7 828 354
7	2 414 682 040 998
8	56 130 437 228 687 557 907 788

Outline

1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Dedekind number $M(n) =$

- the number of antichains of subsets of an n-element set

$$2^{2^{n-\log n}} \leq M(n) \leq 2^{2^{n-\frac{\log n}{3}}}$$

NFA-to-DFA: tight upper bound $= M(n-1)$

- upper bound: show that each set of sets is equivalent to an antichain in DFA for $b(L)$
- tightness for $|\Sigma| = 2^{n+1}$
- conjecture: 6-letter alphabet should work

Outline

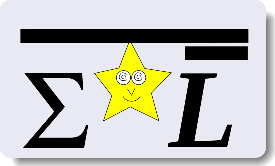
1. Basic notions: DFA, NFA, AFA
2. Forever operator $L \to b(L) = \overline{\Sigma^* \overline{L}}$
3. Birget's results
4. Our improvements and new results
5. Open problems

Birget's results:

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	AFA
DFA	2^{n-1}	3	2^{n-1}	3	
NFA			$\geq 2^{n-1}$ $\leq 2^{n+1}+1$	3	
AFA					$\leq n+1$

Our improvements and new results

$L \backslash b(L)$	DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	AFA	$\|\Sigma\|$
DFA	2^{n-1}	2			n	3
NFA	$M(n-1)$	2^{n+1}	2^{n-1}	3	n	2
AFA					n	1



Summary

$L \backslash \overline{\Sigma^* \overline{L}}$	DFA	$\lvert\Sigma\rvert$	p-DFA	$\lvert\Sigma\rvert$	NFA	$\lvert\Sigma\rvert$	NNFA	$\lvert\Sigma\rvert$	AFA	$\lvert\Sigma\rvert$	BFA	
DFA	2^{n-1}	2	2^{n-1}	2	2^{n-1}	3	2^{n-1}	3	n	3	n	3
p-DFA	$2^{n-1}+1$	4	2^{n-1}	2	2^{n-1}	3	2^{n-1}	3	n	2	n	2
NFA	$M(n-1)$	2^{n+1}	$M(n-1)-1$	2^{n+1}	2^{n-1}	3	2^{n-1}	3	n	2	n	2
NNFA	$\geq M(n-1)$ $\leq M(n)$	2^{n+1}	$\geq M(n-1)-1$ $\leq M(n)-1$	2^{n+1}	$\geq 2^{n-1}$ $\leq 2^n-1$	3	$\geq 2^{n-1}$ $\leq 2^n-2$	3	$n+1$	2	n	2
AFA	$2^{2^{n-1}}$	2	$2^{2^{n-1}}-1$	2	$2^{n-1}+1$	2	2^{n-1}	1	n	1	n	1
BFA	2^{2^n-1}	2	$2^{2^n-1}-1$	2	2^n	2	2^n-1	1	$n+1$	1	n	1

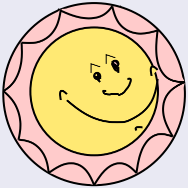
Summary

$L \backslash \overline{\Sigma^* \overline{L}}$	DFA	$\|\Sigma\|$	p-DFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$	NNFA	$\|\Sigma\|$	AFA	$\|\Sigma\|$	BFA	
DFA	2^{n-1}	2	2^{n-1}	2	2^{n-1}	3	2^{n-1}	3	n	3	n	3
p-DFA	$2^{n-1}+1$	4	2^{n-1}	2	2^{n-1}	3	2^{n-1}	3	n	2	n	2
NFA	$\mathrm{M}(n-1)$	2^{n+1}	$\mathrm{M}(n-1)-1$	2^{n+1}	2^{n-1}	3	2^{n-1}	3	n	2	n	2
NNFA	$\geq \mathrm{M}(n-1)$ $\leq \mathrm{M}(n)$	2^{n+1}	$\geq \mathrm{M}(n-1)-1$ $\leq \mathrm{M}(n)-1$	2^{n+1}	$\geq 2^{n-1}$ $\leq 2^n-1$	3	$\geq 2^{n-1}$ $\leq 2^n-2$	3	$n+1$	2	n	2
AFA	$2^{2^{n-1}}$	2	$2^{2^{n-1}}-1$	2	$2^{n-1}+1$	2	2^{n-1}	1	n	1	n	1
BFA	2^{2^n-1}	2	$2^{2^n-1}-1$	2	2^n	2	2^n-1	1	$n+1$	1	n	1

Open problems

- NNFA to {DFA, p-DFA, NFA, NNFA}: tightness
- smaller alphabets

Thank you for your attention



Merci beaucoup

pour votre attention

AFA for $L \to$ DFA for L^R

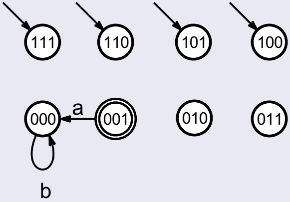
Theorem (Fellah, Jürgensen, Yu 1990)

If L is accepted by an n-state AFA, then
L^R is accepted by a 2^n-state DFA of which 2^{n-1} are final.

Proof idea (AFA to NNFA; $n = 3$):

δ	a	b
$\to q_1$	q_3	q_1
q_2	$q_1 \vee q_2$	q_3
$\circledcirc$ q_3	$\overline{q_1}$	$q_1 \wedge \overline{q_2}$

$\to$



Initial: with first component 1
Final: $f = (0, 0, 1)$
Evaluate $(\delta(q_1, a), \delta(q_2, a), \delta(q_3, a))$ at (0,0,0) ... gives (0,0,1)
Evaluate $(\delta(q_1, b), \delta(q_2, b), \delta(q_3, b))$ at (0,0,0) ... gives (0,0,0)
The reverse of this NNFA is deterministic! □

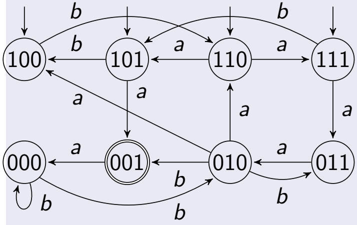
DFA for $L^R \rightarrow$ AFA for L

Theorem (Fellah, Jürgensen, Yu 1990, Jiraskova 2012)

If L^R is accepted by an 2^n-state DFA of which 2^{n-1} are final, then L is accepted by an n-state AFA.

Proof idea (NNFA to AFA; $n = 3$):

L : 2^n-state NNFA, 2^{n-1} initial, one final, rev. det.



$(\{q_1, q_2, q_3\}, \{a, b\}, \cdot, q_1, \{q_3\})$

	$q_1 \cdot a$	$q_2 \cdot a$	$q_3 \cdot a$
000			
001			
010			
011			
100			
101			
110			
111			

□

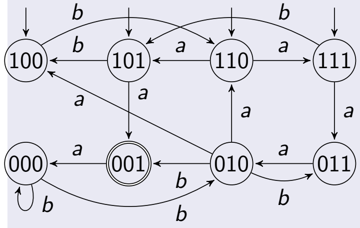
DFA for $L^R \to$ AFA for L

Theorem (Fellah, Jürgensen, Yu 1990, Jiraskova 2012)

If L^R is accepted by an 2^n-state DFA of which 2^{n-1} are final, then L is accepted by an n-state AFA.

Proof idea (NNFA to AFA; $n = 3$):

L : 2^n-state NNFA, 2^{n-1} initial, one final, rev. det.



$(\{q_1, q_2, q_3\}, \{a, b\}, \cdot, q_1, \{q_3\})$

	$q_1 \cdot a$	$q_2 \cdot a$	$q_3 \cdot a$
000	0	0	1
001			
010			
011			
100			
101			
110			
111			

□

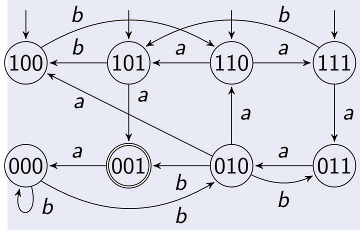
DFA for $L^R \to$ AFA for L

Theorem (Fellah, Jürgensen, Yu 1990, Jiraskova 2012)

If L^R is accepted by an 2^n-state DFA of which 2^{n-1} are final, then L is accepted by an n-state AFA.

Proof idea (NNFA to AFA; $n = 3$):

L : 2^n-state NNFA, 2^{n-1} initial, one final, rev. det.



$(\{q_1, q_2, q_3\}, \{a, b\}, \cdot, q_1, \{q_3\})$

	$q_1 \cdot a$	$q_2 \cdot a$	$q_3 \cdot a$
000	0	0	1
001	1	0	1
010			
011			
100			
101			
110			
111			

□

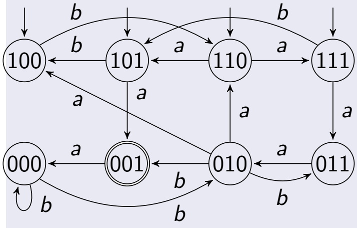
DFA for $L^R \to$ AFA for L

Theorem (Fellah, Jürgensen, Yu 1990, Jiraskova 2012)

If L^R is accepted by an 2^n-state DFA of which 2^{n-1} are final, then L is accepted by an n-state AFA.

Proof idea (NNFA to AFA; $n = 3$):

L : 2^n-state NNFA, 2^{n-1} initial, one final, rev. det.



$(\{q_1, q_2, q_3\}, \{a, b\}, \cdot, q_1, \{q_3\})$

	$q_1 \cdot a$	$q_2 \cdot a$	$q_3 \cdot a$
000	0	0	1
001	1	0	1
010	0		
011	1		
100	0		
101	1		
110	0		
111	1		

□

AFA for $L \to$ DFA for L^R

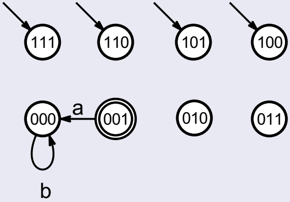
Theorem (Fellah, Jürgensen, Yu 1990)

If L is accepted by an n-state AFA, then L^R is accepted by a 2^n-state DFA of which 2^{n-1} are final.

Proof idea (AFA to NNFA; $n = 3$):

δ	a	b
$\to q_1$	q_3	q_1
q_2	$q_1 \vee q_2$	q_3
$\circledcirc\ q_3$	$\overline{q_1}$	$q_1 \wedge \overline{q_2}$

$\to$



Initial: with first component 1

Final: $f = (0, 0, 1)$

Evaluate $(\delta(q_1, a), \delta(q_2, a), \delta(q_3, a))$ at (0,0,0) ... gives (0,0,1)

Evaluate $(\delta(q_1, b), \delta(q_2, b), \delta(q_3, b))$ at (0,0,0) ... gives (0,0,0)

The reverse of this NNFA is deterministic! □