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REQUIEM AFTER IVAN DOBRAKOV

On September 18, 1997, Ivan Dobrakov passed away in Bratislava. On September 26, a church funeral at the town crematorium took place in a small circle of relatives and members of the mathematical community.

Introitus: Requiem and Kyrie. In the series of seven papers on integration, Ivan Dobrakov built up a Lebesgue type theory of linear integration in Banach spaces.

Sequence: Dies Irae, Tuba Mirum, Rex Tremendae, Recordare, Confutatis, Lacrimosa. Then there are seven papers on multilinear integration in Banach spaces, a natural generalization of the linear theory. Note that there is no "classical scalar theory" of multilinear integration. This theory is much more complicated than the linear and it was developed due to new treatments and techniques from the linear integration.

The whole integration theory of I. Dobrakov is selfcontained, not a set of individual papers about integration. From this viewpoint, his life project has been complete.

The Dobrakov's theory starts from a given operator valued measure $m : P \to L(X,Y)$, where P is a δ-ring of subsets of the set T, X and Y are two Banach spaces, $L(X,Y)$ the space of all bounded linear operators countably additive in the strong operator topology with the finite semivariation $\hat{m}$ on P. There is a bijection between the measures m and the elementary integrals $I : S(P,X) \times \sigma(P) \to Y, I(f,E) = \int_E f\ dm$, where $S(P,X)$ is the space of all P-simple function $f : T \to X$, $\sigma(P)$ is the σ-ring generated by P such that

(1) for every $f \in S(P,X)$ and every $E \in \sigma(P)$, $I(f,E) = I(f, E \cap \{t \in T; f(t) \neq 0\})$;
(2) for a fixed $f \in S(P,X)$, $I(f,\cdot) : \sigma(P) \to Y$ is a countably additive vector measure;
(3) for a fixed $E \in P, I(\cdot,E) : (S(P,X), \|\cdot\|_T) \to Y$ is a continuous linear operator with the norm $\hat{m}(E)$ ($\|\cdot\|_T$ is the supremum norm).

The definition of the Dobrakov integral is based on the following statement.

Theorem *. *Let $f_n : T \to X,\ n = 1,2,\dots,$ be P-simple functions. Let $f : T \to X$ be a P-measurable function, a pointwise limit of a sequence of P-simple functions. Let $f_n(t) \to f(t)$ for m-almost every $t \in T$. Then the following three assertions are mutually equivalent:*

(1) $\lim_{n\to\infty} \int_E f_n\ dm = \nu(E) \in Y$ *exists for every* $E \in \sigma(P)$;
(2) *the integrals* $\int_{\cdot} f_n\ dm : \sigma(P) \to Y,\ n = 1,2,\dots,$ *are uniformly* σ*-additive;*
(3) $\lim_{n\to\infty} \int_E f_n\ dm = \nu(E) \in Y$ *exists uniformly with respect to* $E \in \sigma(P)$.

If (a) or (b) or (c) holds, then $\int_E f\ dm = \nu(E) \in Y$ is unambiguously defined for every f, E, m and the function f is called to be (Dobrakov) integrable.

Denote by $\mathcal{I}(m)$ the space of all Dobrakov integrable functions. The principal trick of the theory is that Theorem * holds also if we replace P-simple f_n with integrable f_n.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Denote by $\hat{m}(g, E)$ the L_1-seminorm of a P-measurable function $g : T \to X$ on $E \in \sigma(P)$, where $\hat{m}(g, E) = \sup\{|\int_E f\ dm|; f \in S(P, X), \|f\|_T \leq 1\}$. The set of all $g : T \to X$ such that $\hat{m}(g, \cdot) : \sigma(P) \to [0, \infty)$ is continuous (i.e. $E_n \downarrow \emptyset \implies \hat{m}(g, E) \to 0$) is a complete seminorm space $\mathcal{L}_1(m)$. In $\mathcal{L}_1(m)$, the strong convergence theorems hold, the analogues of the classical ones, namely the Lebesgue Dominated Convergence Theorem, Vitali Theorem, Monotone Convergence Theorem.

So, we have two types of spaces of integrable functions, $\mathcal{I}(m)$ and $\mathcal{L}_1(m)$, $\mathcal{I}(m) \supset \mathcal{L}_1(m)$. The space $\mathcal{I}(m)$ is a very large space with a group of "nice properties". The strong convergence theorems from $\mathcal{L}_1(m)$ are applied via functionals techniques to proofs in $\mathcal{I}(m)$. In other words, while the classical Lebesgue theory is based on the absolute convergence of series, the theory of $\mathcal{I}(m)$ can be rewritten in the language of unconditional convergence of series in Banach spaces.

Offertorium: Domine Jesu, Hostias. H. B. Maynard proved the general Radon - Nikodým Theorem for Dobrakov integral. C. Schwartz showed that the undefinite integral of a measurable weak integrable function ranks in the second dual of Y. If those values are in the image of Y (we mention the canonical embedding), then the function is Dobrakov integrable. P. Morales proved a Mean Value Theorem for the Dobrakov integral. J. K. Brooks and N. Dinculeanu introduced and considered the space $\mathcal{L}_1(N)$ for a general system of nonnegative measures N and applied their results to the situation when N is the system measures induced with the measure m.

Sanctus. Benedictus. Agnus Dei. There are articles of M. Duchoň, C. Schwartz, R. Chivukula Rao, and A. S. Sastry using the Bartle's integral in the context of locally convex spaces (the spaces of all Bartle and Dobrakov integrable functions coincide in the case of continuous semivariation and considering the same set systems).

S. K. Mitter and S. K. Young showed that without the assumption of the finiteness or σ-finiteness of semivariation, the Dobrakov's integral can be enlarged from the space of all P-simple function to the completion of the tensor product of the space of scalar simple functions and the space X in the projective tensor norm.

Papers of C. Debieve, S. K. Roy with N. D. Charkaborty, W. V. Smith, D. H. Tucker provides some generalizations of some parts of the Dobrakov theory to general locally convex spaces.

Communio: Lux Aeterna. We observe a specific sequence of lives with respect to integration in functional spaces on the Mathematical Institute of the Slovak Academy of Sciences: I. Dobrakov, I. Kluvánek, L. Mišík (in the alphabetical order).

Applying to the present situation, if a renowned composer of music dies suddenly and his work is unfinished, then the difficulties of Süssmayr's situation become clear. Was it his duty as a composer and Mozart's pupil to seek to emulate him? Or should he use his own subjective taste? Mozart wanted to train not just copies of himself but individuals who would stand on their own feet.

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