Foreword

This text was meant as a course text for the subject Functional analysis for English speaking students of the bachelor level in Applied mathematics, Žilina University. The text is a compilation of the sources listed in the bibliography and is certainly not meant to replace numerous good books on the subject. The all material in the present text is standard and my role consisted in the choice, setting a concept, and, hope, in a slight clarification of the material. The text follows the idea of minimality, to give the material via the shortest but precise way.

There are different opinions what is an essence of Functional analysis and what belongs to this subject and what does not. For instance, it is a question whether to include Appendices of this publication or some classical models of Theoretical Physics in the main matter.

My way is to give to the student a serious starting point from what he/she would be able to individually expand and progress in many different directions, to apply methods of functional analysis according to his/her own aims, in particular, to differential equations, models of physics, and engineering.

It has be stressed that an appropriate exercise book is necessarily needed as a complement to this text since mathematics (generally spoken) is thoroughly studied only when solving problems. There exist few serious exercise books in the literature which satisfy excellently. In my teaching, I use the exercise book of P. Quittner, [24].

Chapter 1 is about metric spaces, a structure which serves for constructing a theory of functions using only the set theory and the subset structure called topology. This theory is described in Chapter 2 and 3. The following chapters are about metric spaces equipped with additional structures. The linear (= vector) structure is the basic approximation model of the real word quantities in the human brain (the mathematical manifestation of this abstraction is vector spaces); mathematical geometrical structures manifest the form abstractions (inner product defines angles and volume); idea of controllers, creation, change, action is manifested in the notion of operator; interplay outer-inner, physical-spiritual – notion of duality, and finally ... beauty is manifested in spectral theorems (but not only). Excuse me for expressing these few non-mathematical claims without any detailed explanation, but some nice mystification is allowed in forewords. My reason to do this is to realize that there are also deeper levels than the visible non-emotional formalization so typical for mathematics.

Chapter 4 contains definitions of more rich structures than are used in metric space theory (Chapters 1, 2, 3), relation among notions, examples, and tools how to create new spaces of functions from given ones.

Chapter 5 is about linear operators, in particular, functionals. The goal here is to reach understanding duality. Duality has the extremal importance for the study of general vector spaces, in particular, the locally convex topological vector spaces. However, we basically deal with the Hilbert and Banach spaces.

In Chapter 6, there are fundamental classical theorems.

In Chapter 7, there is a practical consideration of duality, weak, and weak* topologies.

Chapter 8 represent an introduction to compact operators, the important and, in the same time, rather the most useful and easy class of operators in Hilbert space. This is also an introduction to the spectral theory.

I mean that spectral theorems are very beautiful and also useful for mathematics and applications. In Chapter 9, there are considered questions about spectral theory for general settings via expanding to Banach spaces. On the other hand, we deal about restrictions of this general setting to richer structures, Banach algebras and self-adjoint operators in Hilbert spaces.

In Appendices we bring basic information about topics which are preliminaries for the study of functional analysis but not always properly included into the university programs: Vector spaces, Topology, Hausdorff spaces, Overview of topological vector spaces, Lebesgue integration, L^p spaces, and some functional inequalities. The reason to append the text is some convenience of the readers. Students may not be familiar with these topics for the absence of the corresponding courses. Practically for our purposes, we need knowledge of these concepts only on the general level and for overview.

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I plan to translate (and improve) these publication also to the Slovak language since such a basic text about Functional analysis for students of mathematics is absent in the university literature in Slovakia. Please, do not hesitate contact the author for further improvements, picking out misspelling, misprints, finding errors. Thank you very much in advance.

Žilina, September 1, 2008 Ján Haluška

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