Measuring dispersed sunlight

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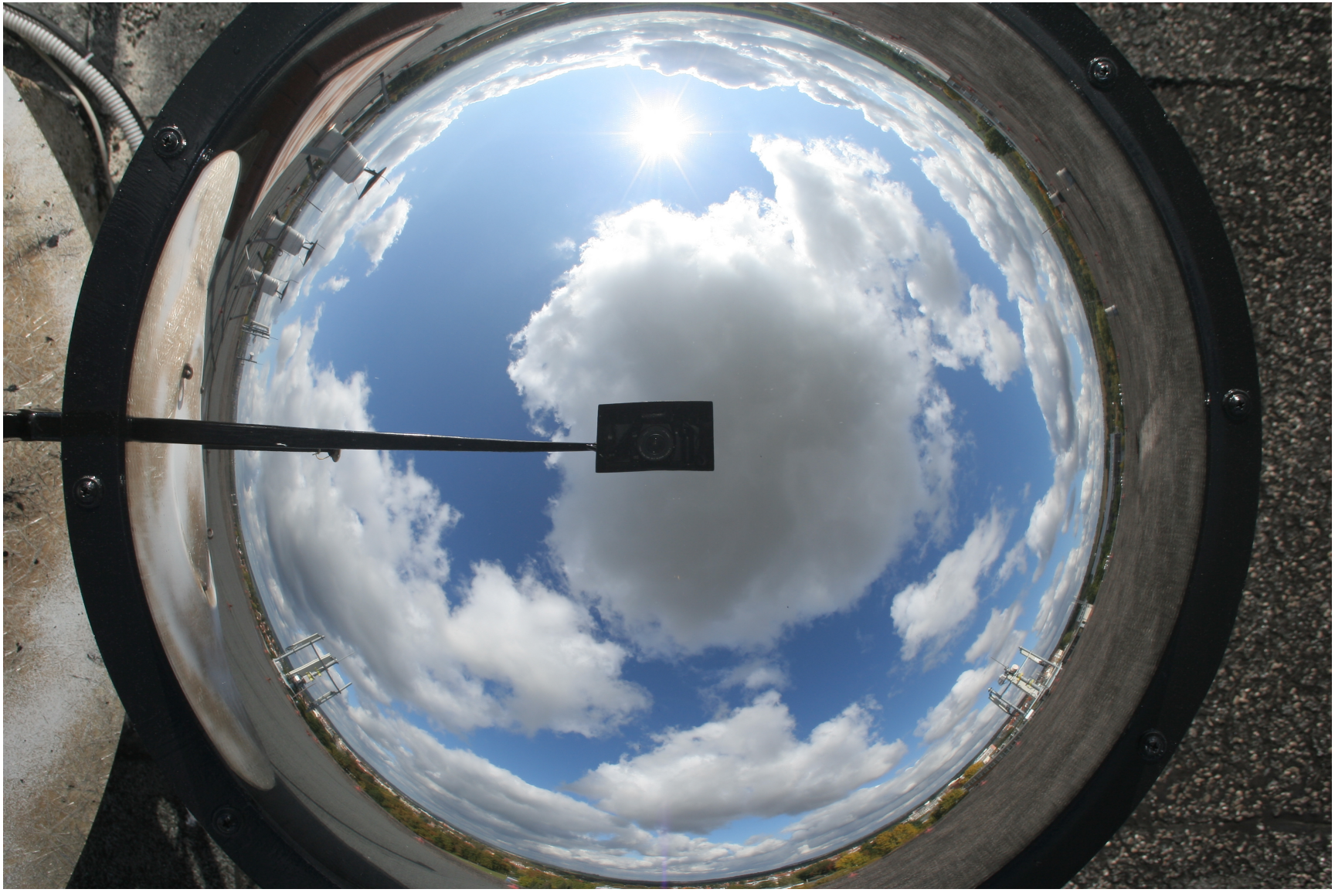
Pyranometer

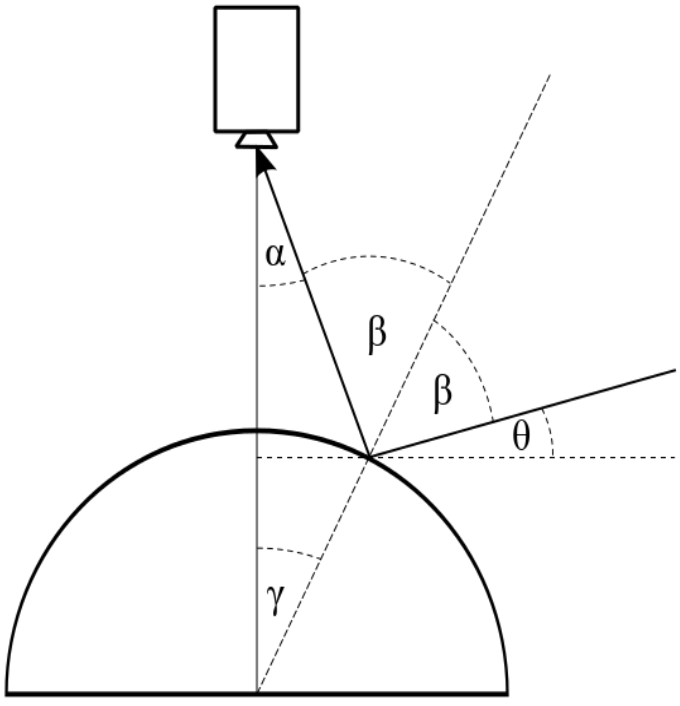
Mirror sphere



Panorama

View from 50°7′47.941″N 14°22′24.202″E

Conversion formula



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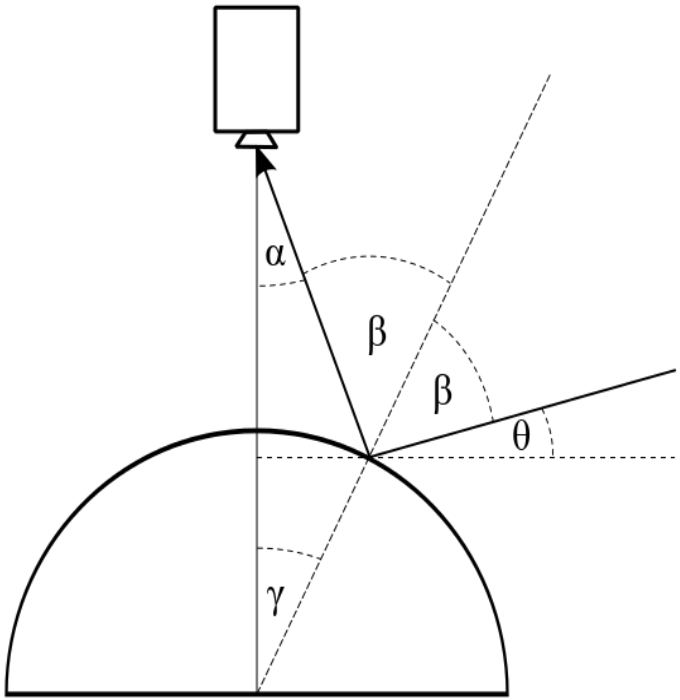

$$(90^\circ - \gamma) + (90^\circ - \alpha) + \beta = 180^\circ$$
$$\Rightarrow \beta = \alpha + \gamma$$

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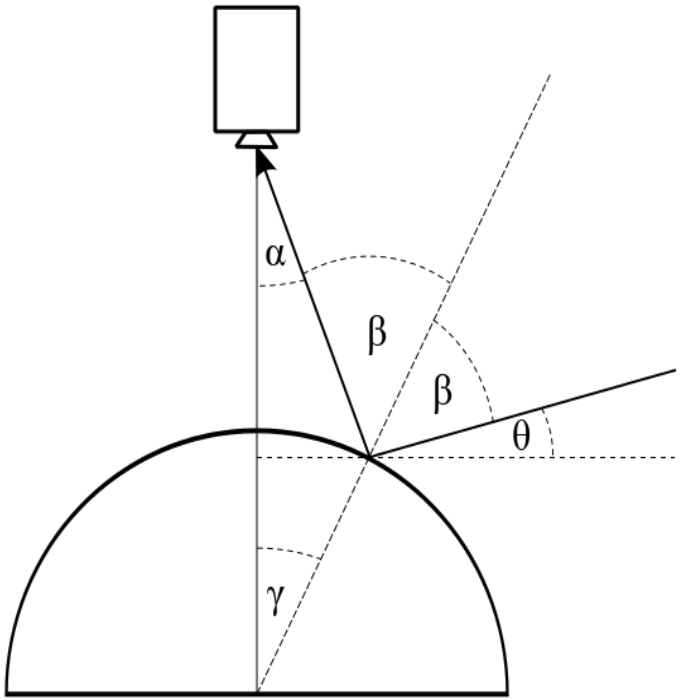

$$(90^\circ - \gamma) + (90^\circ - \alpha) + \beta = 180^\circ$$
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$$(90^\circ - \alpha) + 2\beta + \theta = 180^\circ$$
$$\Rightarrow \theta = 90^\circ - 2\gamma - \alpha$$

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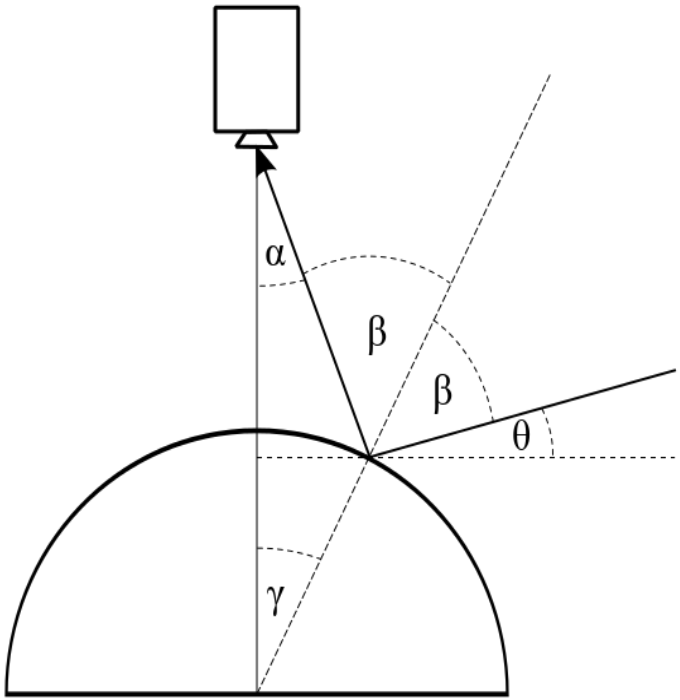

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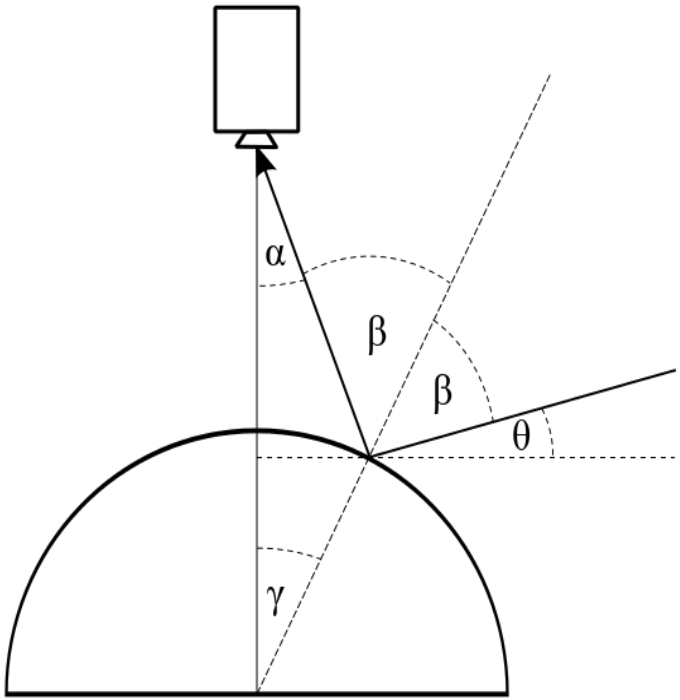
$$\sin\gamma = \frac{x}{r}$$

Conversion formula



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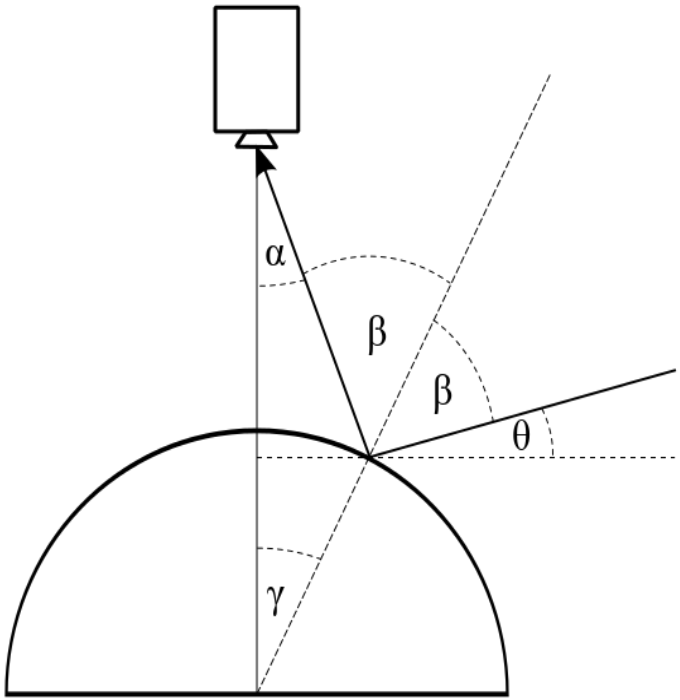

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$$x^2 + y^2 = r^2$$
$$y = kx + h$$
$$\text{where } k = -\cot\alpha$$

Conversion formula (contd.)

$$x^2 + (kx + h)^2 = r^2$$

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$$x^2 + (kx + h)^2 = r^2$$

$$(1 + k^2)x^2 + 2khx + h^2 - r^2 = 0$$

Conversion formula (contd.)

$$x^2 + (kx + h)^2 = r^2$$

$$(1 + k^2)x^2 + 2khx + h^2 - r^2 = 0$$

$$x = \frac{-2kqr \pm \sqrt{4r^2 \cdot (1 + k^2 - q^2)}}{2(1 + k^2)}$$

where $q = \frac{h}{r}$

Conversion formula (contd.)

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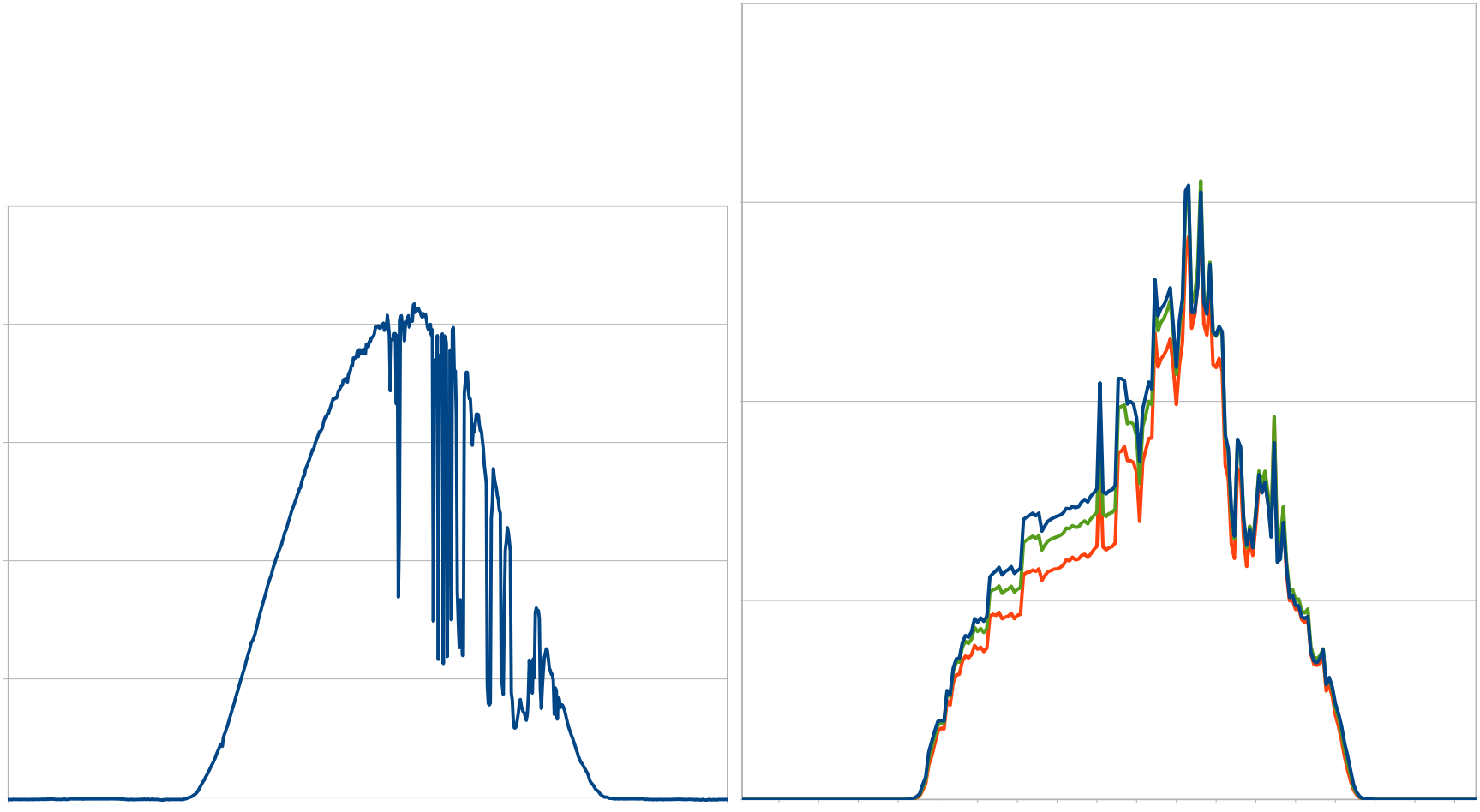
$$\text{where } q = \frac{h}{r}$$

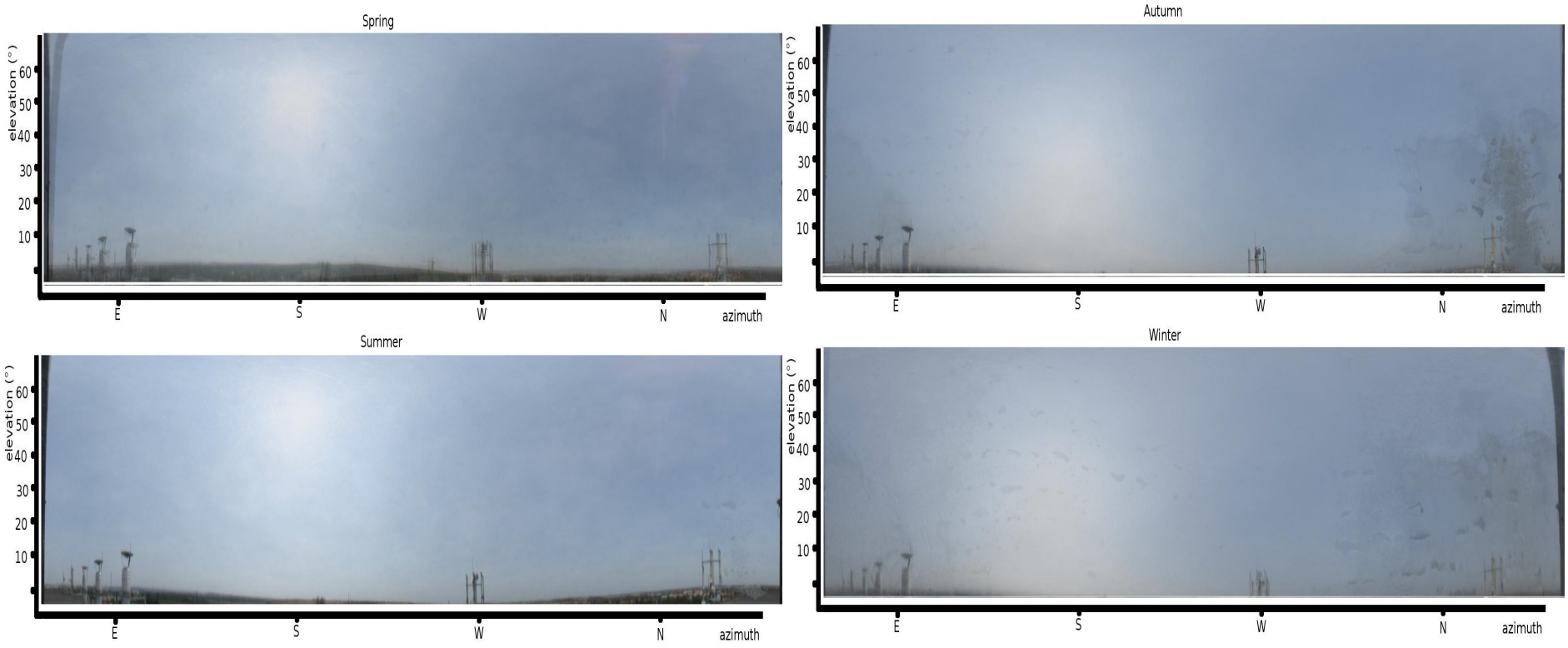
Finally, we use $1 + k^2 = \sin^{-2} \alpha$ and $\gamma = \arcsin \frac{x}{r}$ to obtain

$$\theta = 90^\circ - \alpha - 2 \cdot \arcsin \left(\sin \alpha \cdot \left(q \cos \alpha - \sqrt{1 - q^2 \sin^2 \alpha} \right) \right)$$

Comparing pyranometer and camera

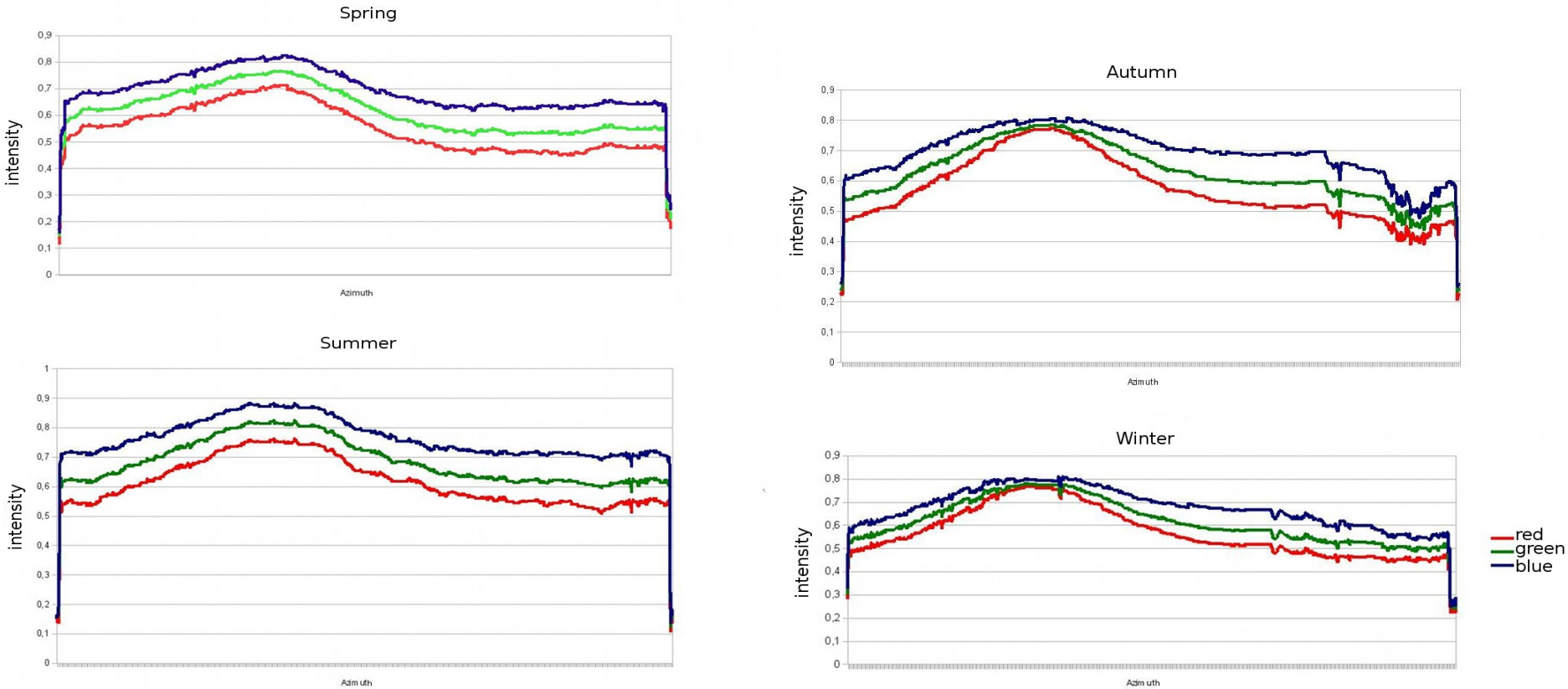


Average skies



We take images of the noon sky and compute their average.

Average skies



We then look 20° above the horizon.