On primeness of cube terms

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joint work with L. Barto

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The lattice of interpretability types

Interpretation from variety $\mathcal{V}$ to variety $\mathcal{W}$ is

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The resulting order is lattice order, let's denote it $\mathcal{L}$ (Garcia, Taylor, The lattice of interpretability types of varieties, 1984).

Joins in $\mathcal{L}$

Join of two varieties $\mathcal{V}$ and $\mathcal{W}$ in $\mathcal{L}$ can be described as the variety $\mathcal{V} \vee \mathcal{W}$ whose operations are operations of both varieties (taken as a discreet union of operations of $\mathcal{V}$ and operations $\mathcal{W}$), and whose indentities are all identities of both varieties.

In the other worlds, we can describe algebras in $\mathcal{V} \vee \mathcal{W}$ as $(A, F \cup G)$ where $(A, F) \in \mathcal{V}$ and $(A, G) \in \mathcal{W}$.

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Example (Malcev $\vee$ majority)

Let $\mathcal{V}$ be variety with single Maltsev operation $q(x, x, y) = q(y, x, x) = y$, and $\mathcal{W}$ be variety $\mathcal{W}$ with majority operation $x = m(x, x, y) = m(x, y, x)$ $= m(y, x, x)$.

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$$p(x, y, z) = q(x, m(x, y, z), z)$$

is a Pixley term $(p(x, x, y) = p(y, x, x) = p(y, x, y) = y)$ of $\mathcal{V} \vee \mathcal{W}$.
On the other hand, Pixley term implies both majority, and Malcev term.
So, Pixley is the join of Malcev and majority.

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Prime

- Groups, cyclic terms of prime arity (Garcia, Taylor, 1984),
- CP (Tschantz, unpublished).

Cube terms

Definition

An *n-cube term* is $(2^n - 1)$-ary term q satisfying equations

$$q \begin{pmatrix} y & x & y & \dots & y \\ x & y & y & \dots & y \\ x & x & x & \dots & y \\ \vdots & & & \ddots & \vdots \\ x & x & x & \dots & y \end{pmatrix} = \begin{pmatrix} x \\ \vdots \\ x \\ x \\ x \end{pmatrix}$$

where the matrix on the left hand side composes of all columns of x's and y's except the one with all x's.

Having a cube term is equivalent to having edge term (BIMMVW, 2010), parallelogram term (Kearnes, Szendrei, 2012).

Coloring of terms by variables

(Sequeira, Barto) Let X be a given set of variables, and $A \subseteq \mathrm{Eq}(X)$. We say that variety $\mathcal{V}$ is A-colorable if there is a map $c \colon F_{\mathcal{V}}(X) \to X$ such that

1. $c(x) = x$ for all $x \in X$, and
2. for every $\alpha \in A$ whenever $f \sim_{\hat{\alpha}} g$ then $c(f) \sim_{\alpha} c(g)$

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In particular, if $\mathcal{V}$ has a Malcev term q then from 1, $c(q) = z$, and from 2, $c(q) = x$. So, $\mathcal{V}$ is not A-colorable.

And, the converse is also true, i.e., variety is CP if and only if it is not A-colorable.

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- congruence n-permutability
  for $A = \{x_0x_1|x_2x_3|\dots x_n, x_0|x_1x_2|x_3\dots x_n\}$,

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- congruence n-permutability for $A = \{x_0x_1|x_2x_3|\dots x_n, x_0|x_1x_2|x_3\dots x_n\}$,
- satisfying non-trivial congruence identity for $A = \{xy|zw, xz|yw, x|yzw\}$.

Coloring for cube terms

Let n be fixed, and let $X = \{x_1, \ldots, x_{2^n-1}\}$, and for $i < n$ define α_i as the equivalence on X defined as $x_k \sim_{\alpha_i} x_l$ if and only if k and l has the same digit in the binary expansion at the i-th position.

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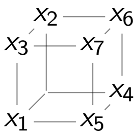
If $n = 3$ then $X = \{x_1, \dots, x_7\}$, $\alpha_0 = x_1x_3x_5x_7|x_2x_4x_6$,
$\alpha_1 = x_1x_4x_5|x_2x_3x_6x_7$, and $\alpha_3 = x_1x_2x_3|x_4x_5x_6x_7$.



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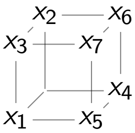

Observation

The n-cube term satisfies coloring condition for $A = \{\alpha_0, \ldots, \alpha_{n-1}\}$.

Linear varieties

Theorem (Sequeira, (Barto); Bentz-Sequeira; O)

Congruence modularity, n-permutability, satisfying non-trivial congruence identity, and n-cube term are prime with respect to varieties axiomatized by linear equations.

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Proof.

Suppose that $\mathcal{V}$ is A-colorable ($A \subseteq \operatorname{Eq} X$). Then there is a structure of $\mathcal{V}$ algebra $\mathbf{X}$ on X such that all $\alpha \in A$ are congruences of $\mathbf{X}$.

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- these operations satisfy all linear identities of $\mathcal{V}$,
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Now, if neither of $\mathcal{V}$, $\mathcal{W}$ satisfy $\mathcal{P}$ then both are A-colorable. So, in both varieties we have algebra on X with congruences A, hence it is in $\mathcal{V} \vee \mathcal{W}$. And we get a coloring of $\mathcal{V} \vee \mathcal{W}$ as the unique expansion of the identity map on X to homomorphism $f\colon F(X) \to \mathbf{X}$. □

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Theorem (Barto, O)

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Cube term blockers

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Theorem (Marković, Maróti, McKenzie, 2012; Barto, Kozik, Stanovský, 2014)

A finite idempotent algebra has a cube term if and only if no subalgebra of **A** *has a cube term blocker.*

The inflating trick

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Conjecture

An n-cube term gives a prime Maltsev filter.

One more observation. . .

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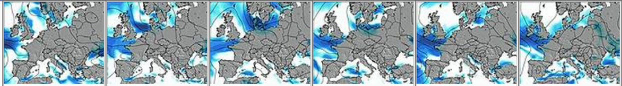
Lomnicky Stit, Slovakia Weather Forecast

Weather Forecast Issued (local time): 1 am Sunday 07 Sep 2014

Days 0-3 Lomnicky Stit Weather Summary: Heavy rain (total 33mm), heaviest during Tue afternoon. Mild temperatures (max 6°C on Mon afternoon, min 4°C on Tue night). Wind will be generally light.

Days 3-6 Lomnicky Stit Weather Summary: Moderate rain (total 16mm) heaviest on Wed afternoon, then becoming colder with a dusting of snow on Fri morning. Freeze-thaw conditions (max 4°C on Wed morning, min 0°C on Sat morning). Wind will be generally light.

Metric	Sunday 7			Monday 8			Tuesday 9			Wednesday 10			Thursday 11			Friday 12		
Imperial	AM	PM	night	AM	PM	night	AM	PM	night	AM	PM	night	AM	PM	night	AM	PM	night
See all weather maps																		
																		
Wind (km/h)	10	10	10	10	10	10	10	15	20	15	20	20	5	5	5	5	10	10
Summary	rain shwrs	rain shwrs	some clouds	clear	rain shwrs	clear	rain shwrs	rain shwrs	rain shwrs	rain shwrs	rain shwrs	rain shwrs	snow shwrs	light rain	light rain	light snow	snow shwrs	some clouds
Snow (cm)	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	2	1	-
Rain (mm)	1	3	-	-	8	-	2	10	9	2	5	1	-	5	3	-	-	-
High °C	5	5	5	5	6	6	5	6	5	4	4	3	3	2	3	2	1	2
Low °C	5	5	5	5	6	6	5	5	4	4	4	3	2	2	2	1	1	1
Chill °C	3	3	3	2	4	4	2	3	2	1	0	0	0	2	3	2	1	0
Freezing level (m)	3400	3450	3500	3450	3550	3550	3450	3450	3400	3250	3300	3200	2900	3100	3100	2750	2900	2900

Source: **http://www.mountain-forecast.com**

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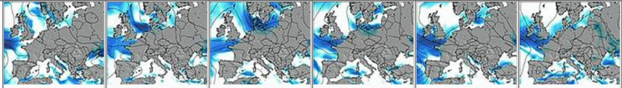
See all weather maps																		
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Summary	rain shwrs	rain shwrs	some clouds	clear	rain shwrs	clear	rain shwrs	rain shwrs	rain shwrs	rain shwrs	rain shwrs	rain shwrs	snow shwrs	light rain	light rain	light snow	snow shwrs	some clouds
Snow (cm)	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	2	1	-
Rain (mm)	1	3	-	-	8	-	2	10	9	2	5	1	-	5	3	-	-	-
High °C	5	5	5	5	6	6	5	6	5	4	4	3	3	2	3	2	1	2
Low °C	5	5	5	5	6	6	5	5	4	4	4	3	2	2	2	1	1	1
Chill °C	3	3	3	2	4	4	2	3	2	1	0	0	0	2	3	2	1	0
Freezing level (m)	3400	3450	3500	3450	3550	3550	3450	3450	3400	3250	3300	3200	2900	3100	3100	2750	2900	2900

Source: **http://www.mountain-forecast.com**

Thank you for your attention!