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Posets of principal lattice congruences

Gábor Czédli (SSAOS 52, Stara Lesna, Sept. 6–12, 2014)

September 6, 2014

Grätzer, G.: The order of principal congruences of a bounded lattice. Algebra Universalis **70**, 95–105 (2013)

If P is bounded, then $\exists L$ such that $P \cong \mathrm{Princ}(L)$.

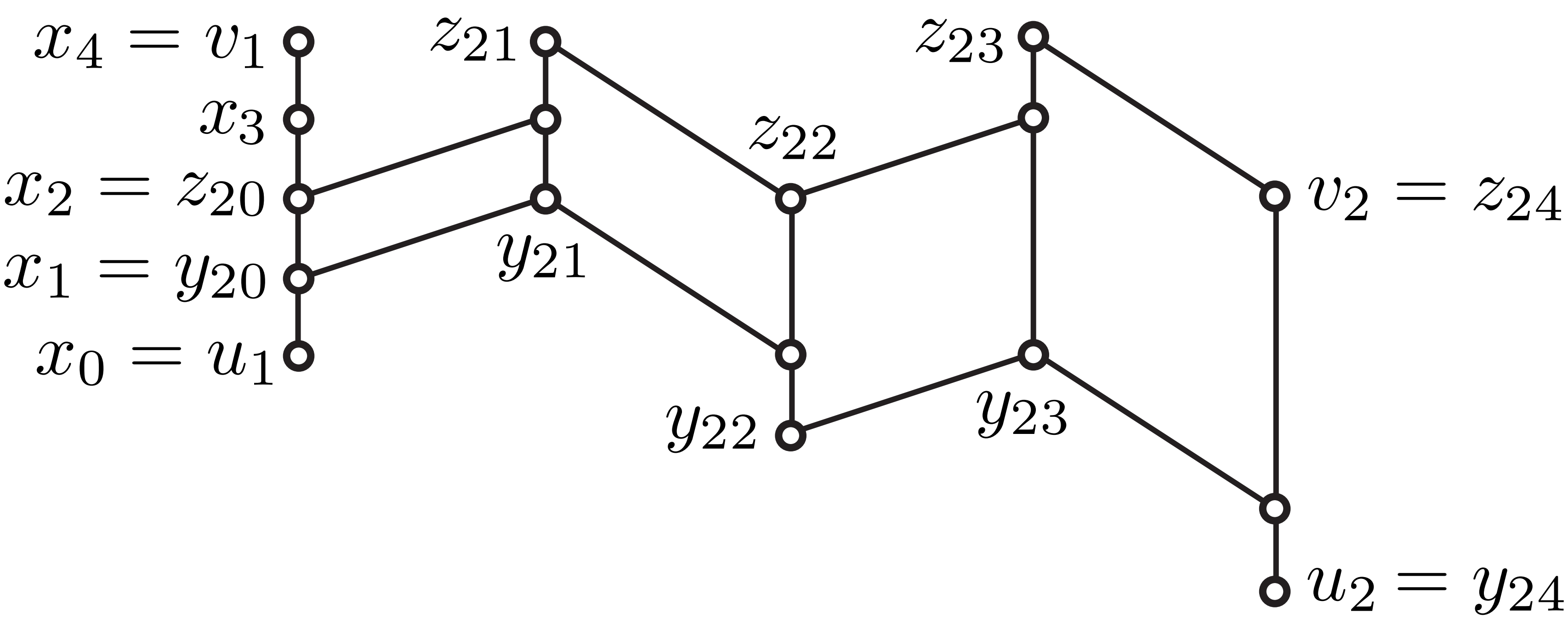
Czédli, G.: Representing homomorphisms of distributive lattices as restrictions of congruences of rectangular lattices. AU **67**, 313–345 (2012)

Gillibert, P.; Wehrung, F.: From objects to diagrams for ranges of functors

Czédli, G.: The ordered set of principal congruences of a countable lattice; submitted to AU

If P has 0, it is directed, and $|P| \leq \aleph_0$, then $\exists L$ such that $P \cong \text{Princ}(L)$.

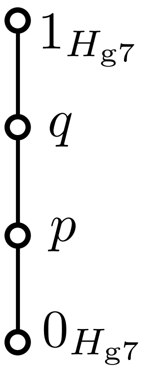
“Fence” (how do congruences spread):



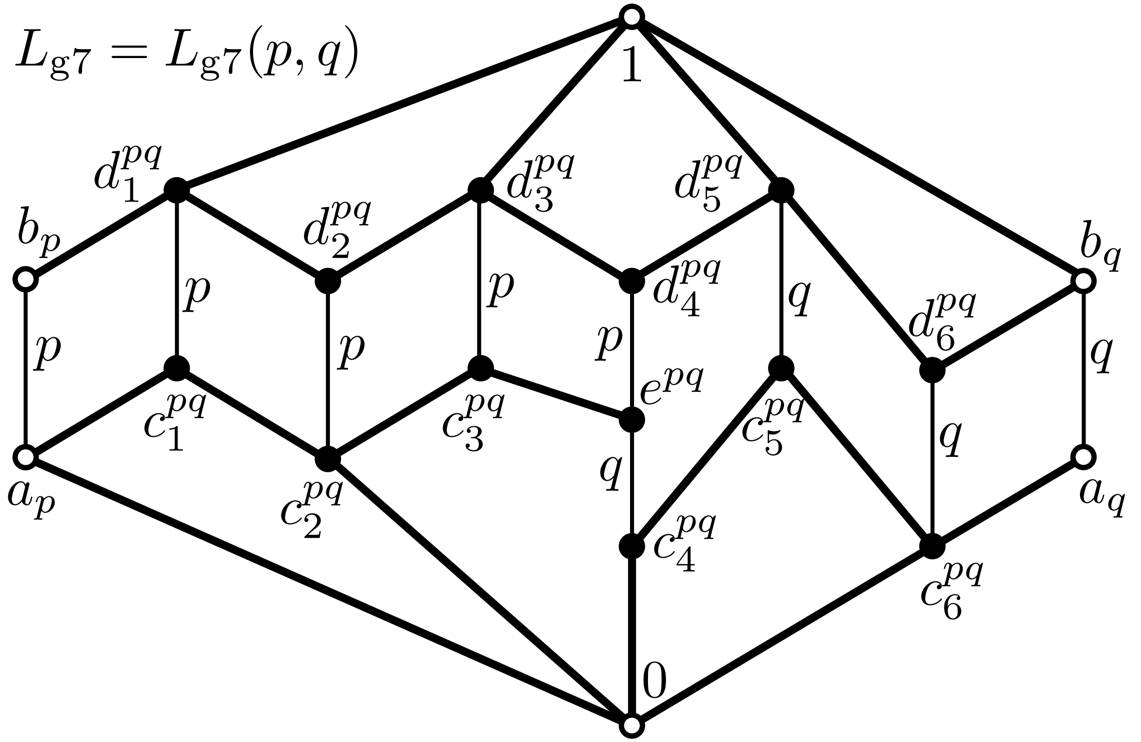
Gadget:

$\mathcal{L}_{\mathrm{g}7} = \mathcal{L}_{\mathrm{g}7}(p, q)$

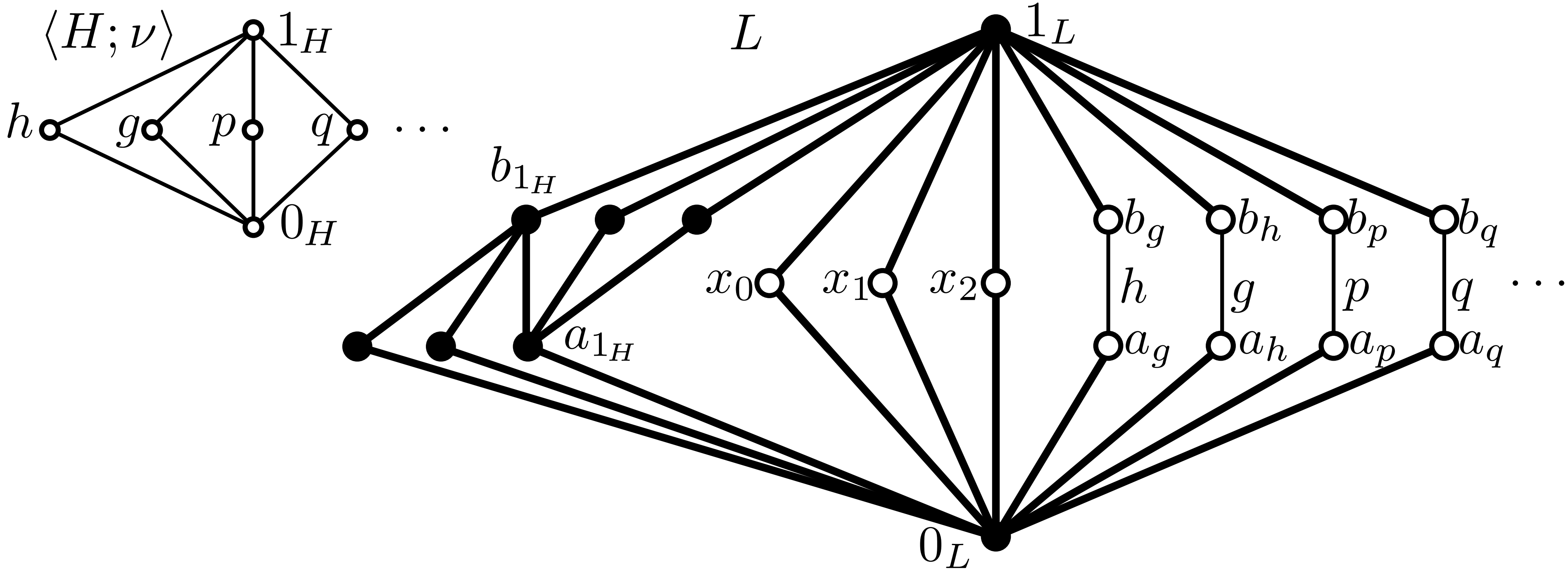
$H_{\mathrm{g}7} = H_{\mathrm{g}7}(p, q)$



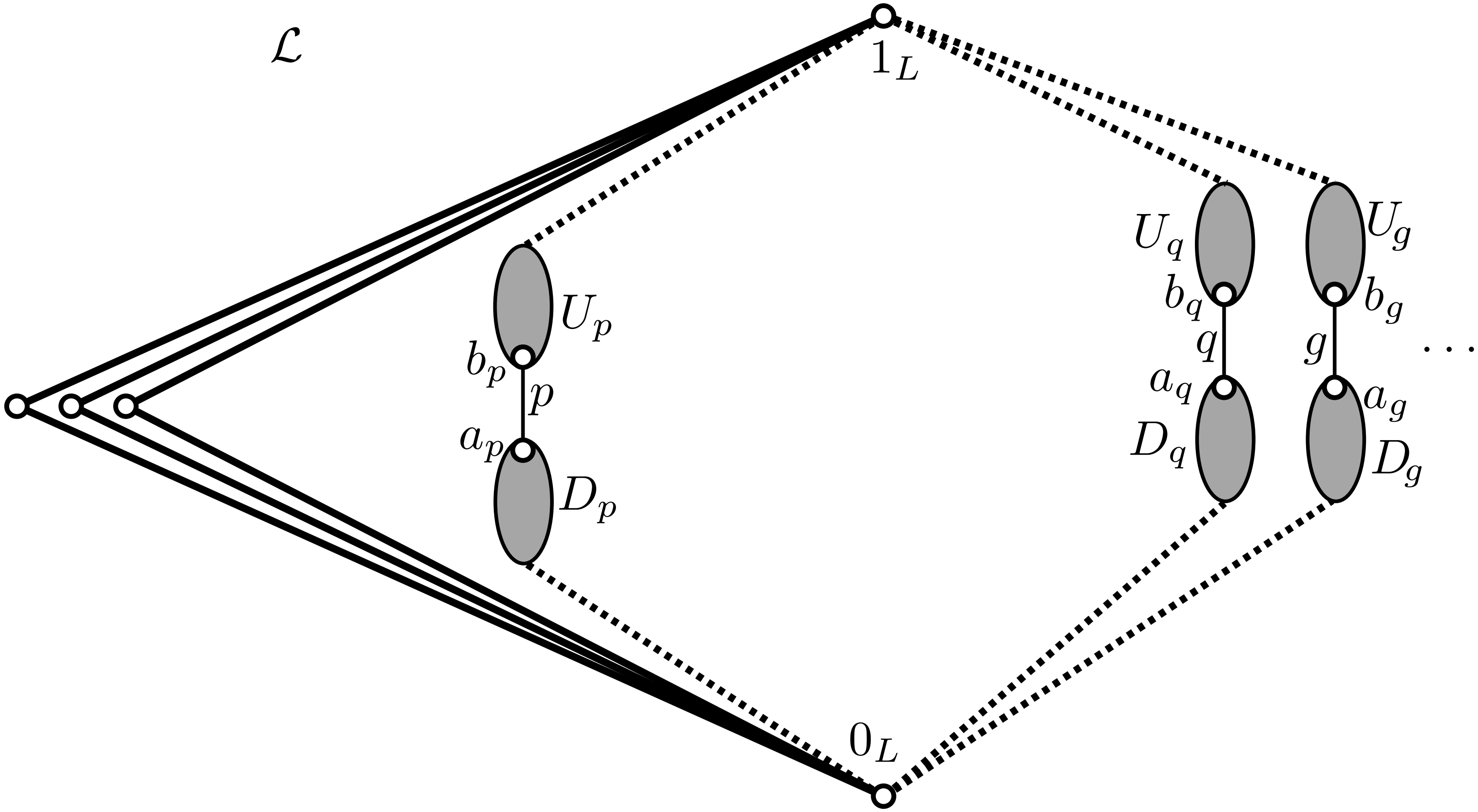
$L_{\mathrm{g}7} = L_{\mathrm{g}7}(p, q)$



If P is a modular lattice with length 2:

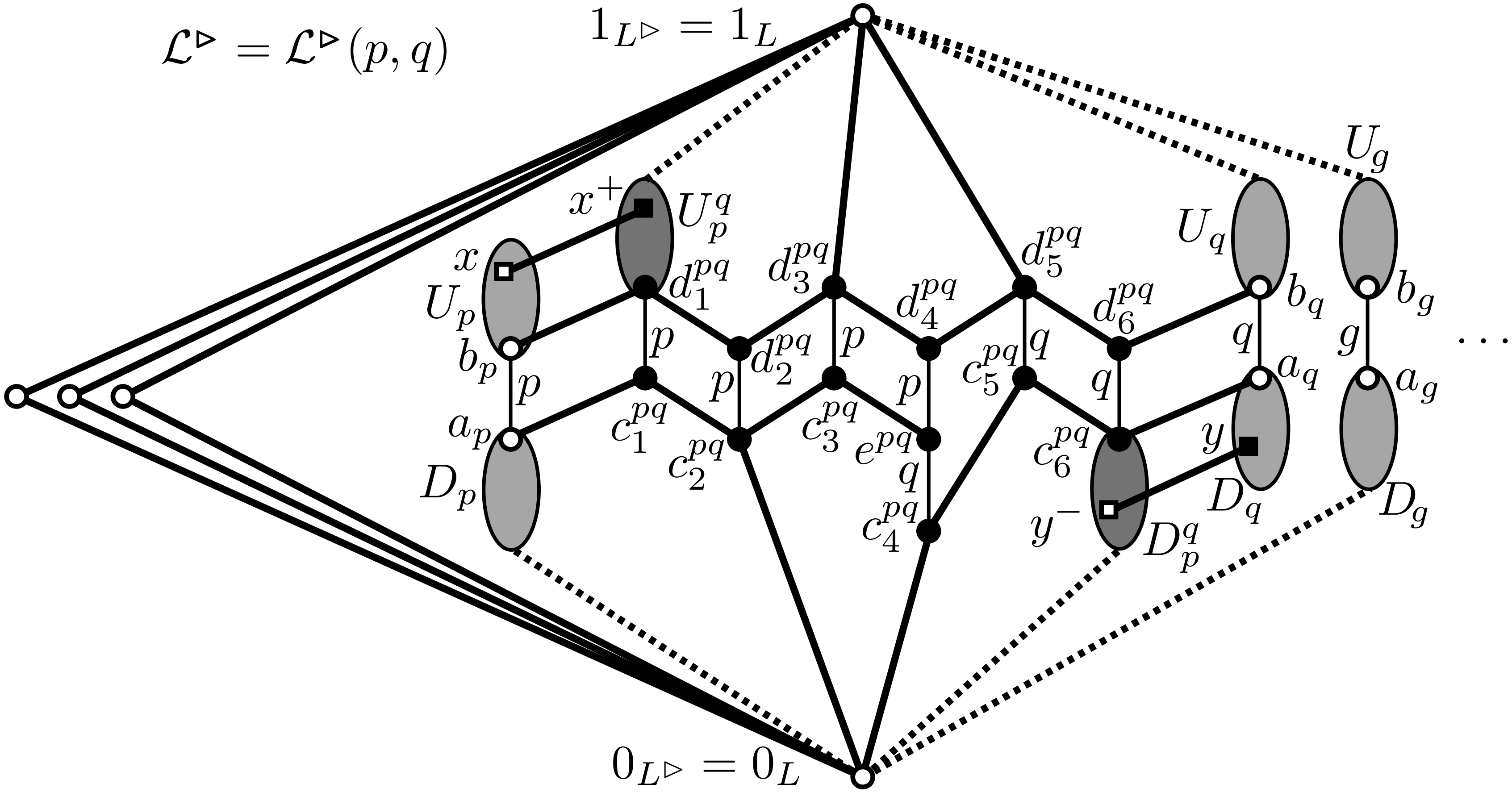


Before a 1-step horizontal extension:

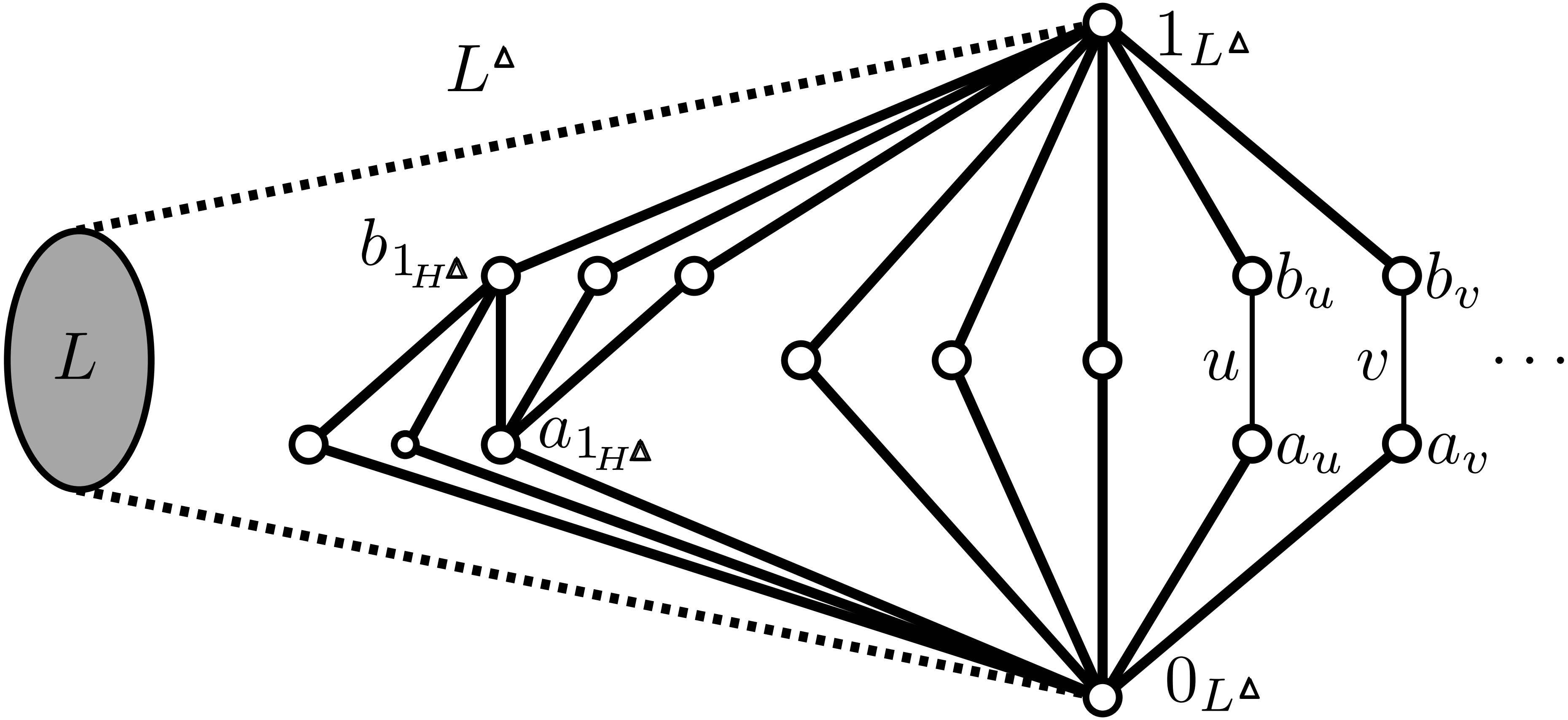


After the 1-step horizontal extension:

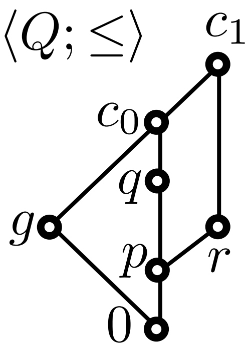
$\mathcal{L}^{\triangleright} = \mathcal{L}^{\triangleright}(p, q)$

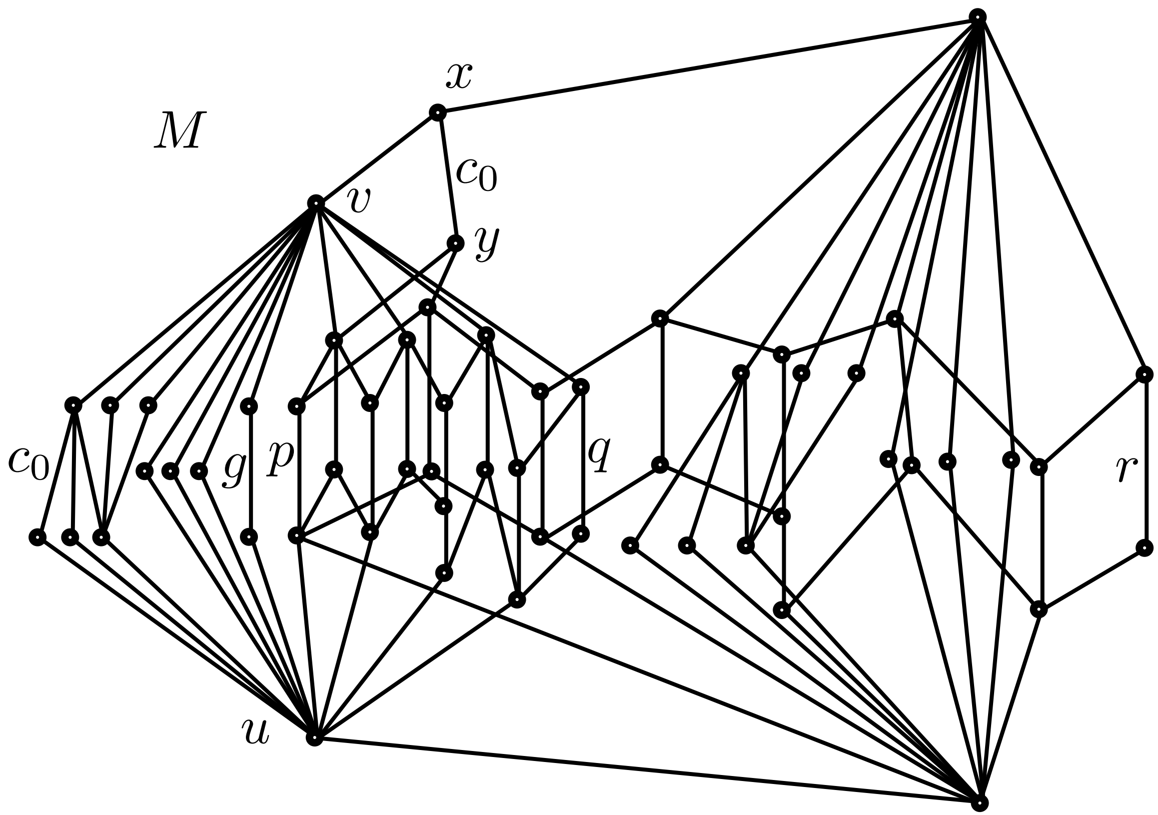


A vertical extension:

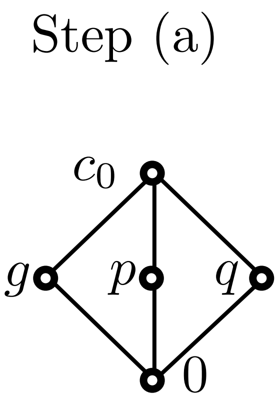


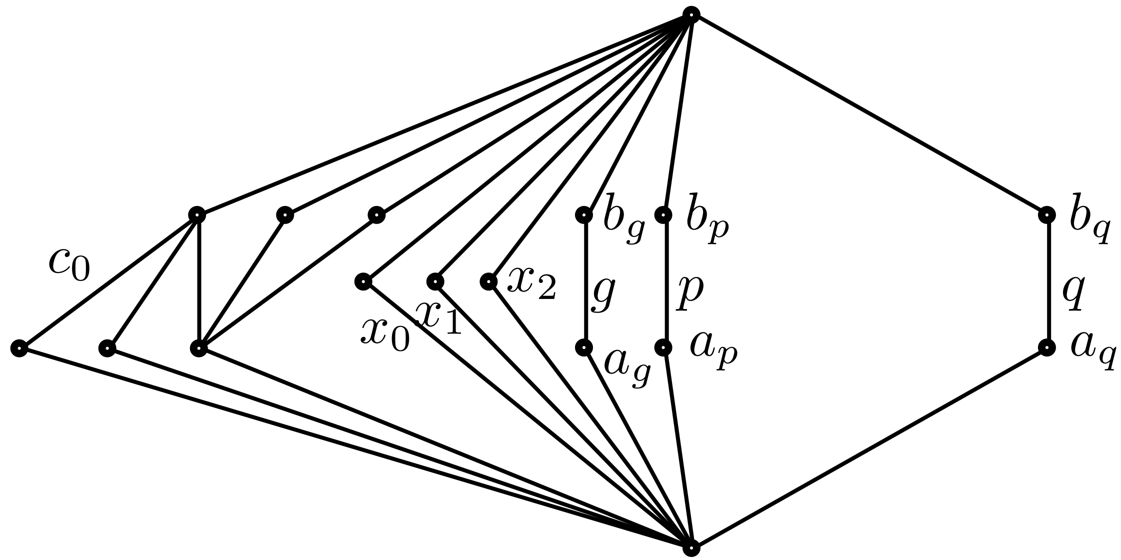
An example (we pretend as if Q had no top):





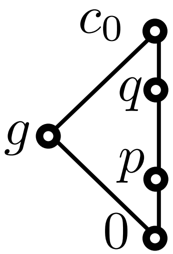
First we go only up to c_0 (but $p \not\leq q$ now):

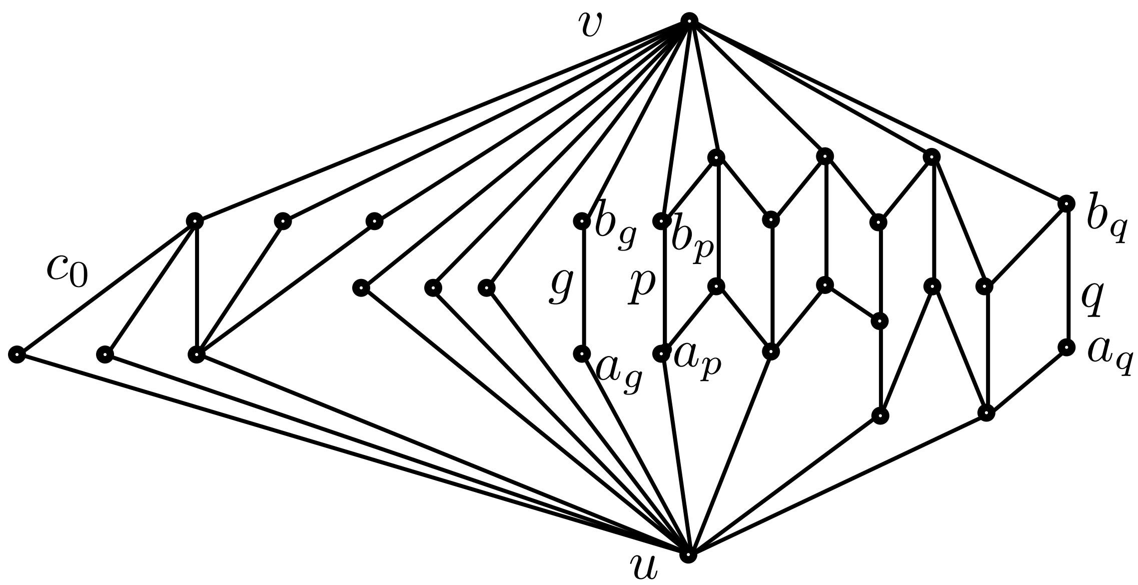




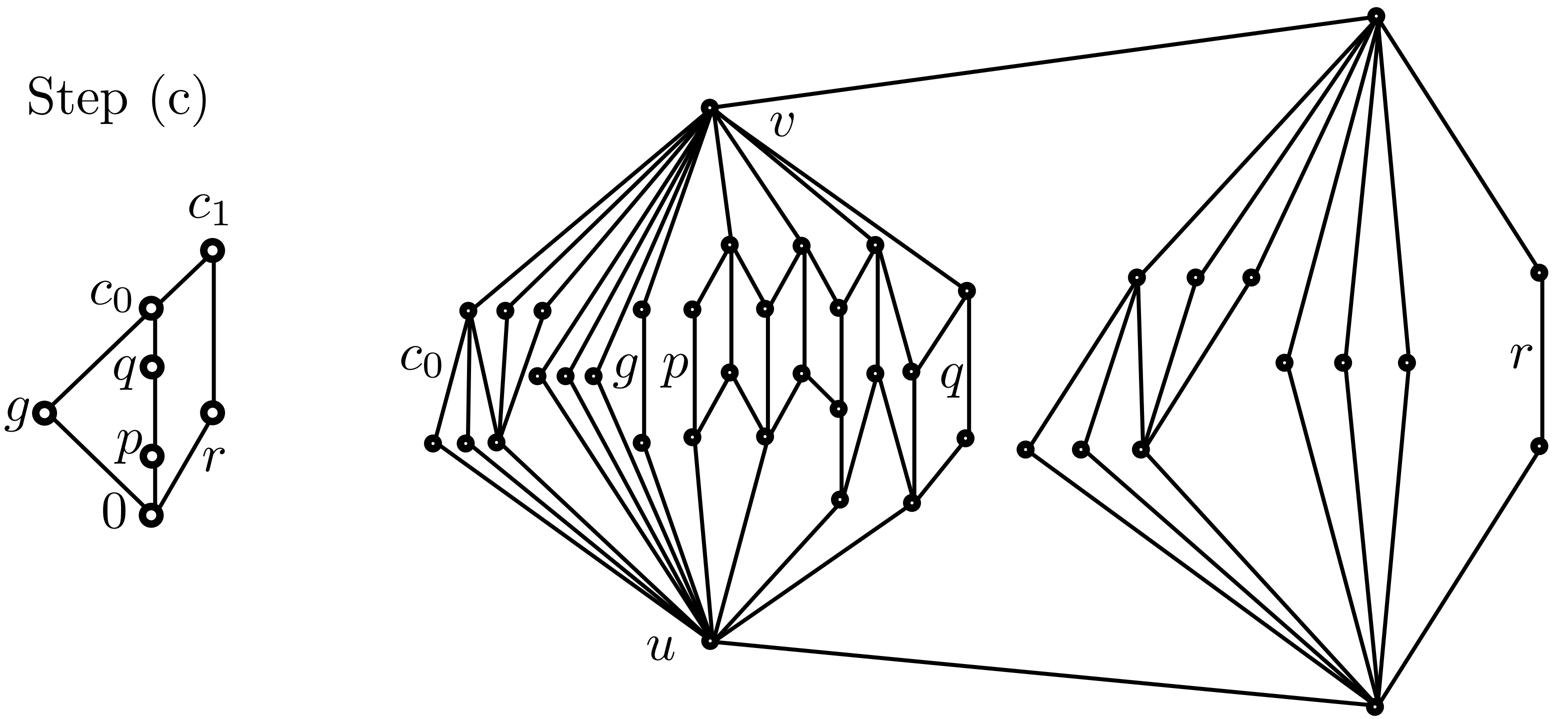
So we need a horizontal 1-step to ensure $p < q$:

Step (b)





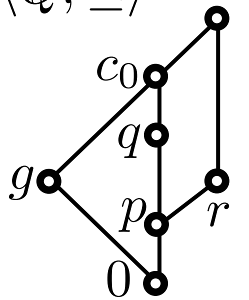
In a vertical step, we add a new top, c_1, and every "new" element below it :



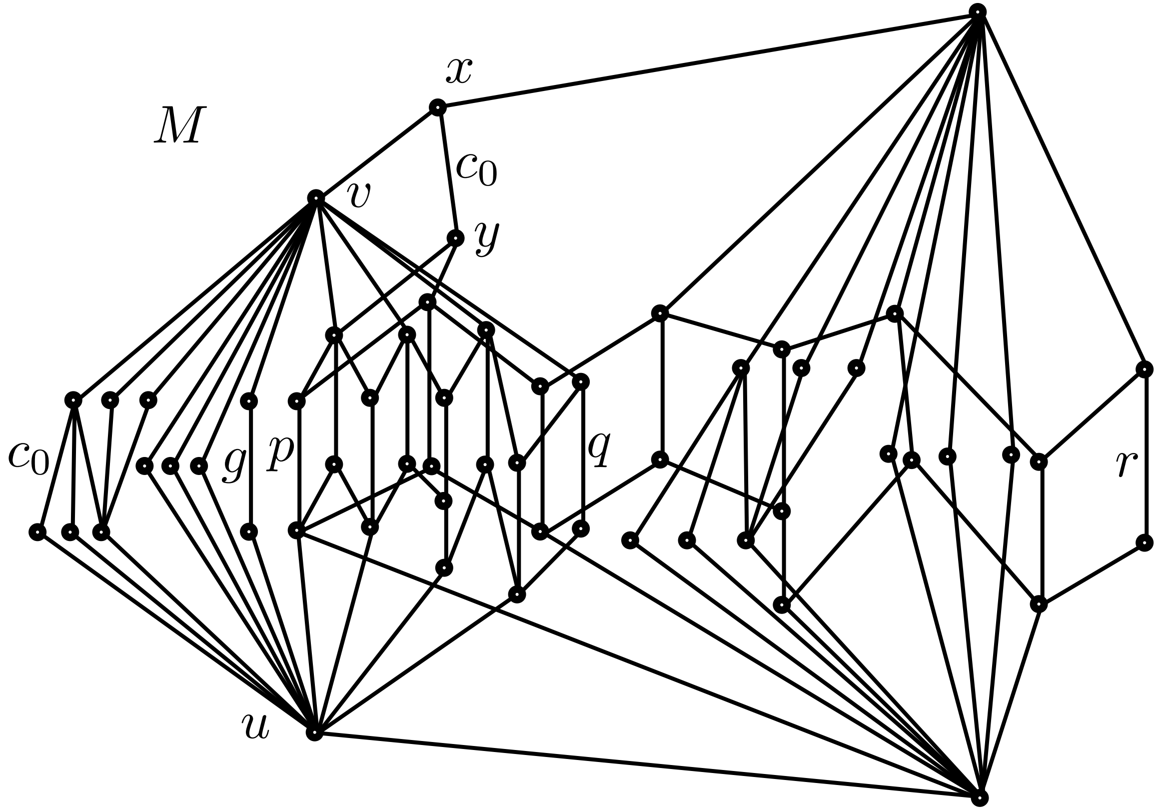
A horizontal 1-step forces $p < r$, and we are ready:

Step (d)

$\langle Q; \leq \rangle$



M



Czédli, G.: Representing some families of monotone maps by principal lattice congruences; arXiv

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