Project 1636/03/01

Kurzweil's and Dobrakov's approaches to integration

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Programme SASPRO
Mobility Programme of the Slovak Academy of Sciences

- November 2015 -

Integration theory: History at a glance

- Bernhard Riemann
- Henri Lebesgue

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Kurzweil integral - Czech Academy of Sciences
J. Kurzweil (1957) - R. Henstock (1960)

J. Kurzweil - seminar in 2009

Integration theory: History at a glance

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- Henri Lebesgue

Kurzweil integral - Czech Academy of Sciences

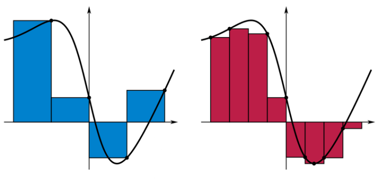
Dobrakov integral - Slovak Academy of Sciences

J. Kurzweil - seminar in 2009

I. Dobrakov

Kurzweil integral

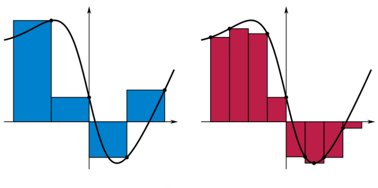
Riemann type, non-absolute integral



$$\int_a^b f \approx \sum f(\xi_j)(t_j - t_{j-1})$$

Riemann $t_j - t_{j-1} < \delta \longrightarrow$ Kurzweil $t_j - t_{j-1} < \delta(\xi_j),\ \ \xi_j \in [t_{j-1}, t_j]$

Kurzweil integral



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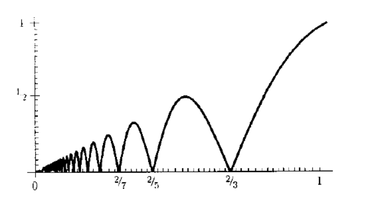
$$t_j - t_{j-1} < \delta(\xi_j)$$

Example:

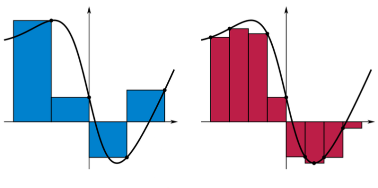
$$f(t) = F'(t)$$

where

$$F(t) \begin{cases} t\,|\cos(\pi/t)|, & t \in (0,1], \\ 0, & t = 0 \end{cases}$$



Kurzweil integral



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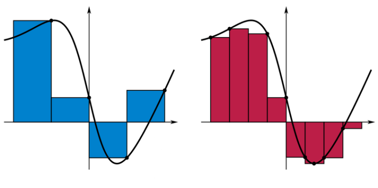

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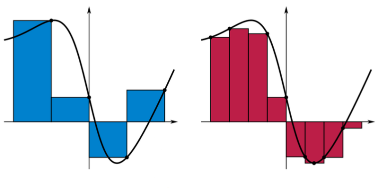
Kurzweil-Stieltjes integral: $f, g : [a, b] \to \mathbb{R}$

$$\int_a^b f \, \mathrm{d}g \approx \sum f(\xi_j)\big(g(t_j) - g(t_{j-1})\big) \qquad t_j - t_{j-1} < \delta(\xi_j)$$

Kurzweil integral



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Application:

- dynamic equations on time scales
- equations with impulses
- functional differential equations

abstract Kurzweil-Stieltjes integral: Banach space X

$$F : [a, b] \to \mathcal{L}(X) \text{ and } g : [a, b] \to X \text{ define } \int_a^b F \,\mathrm{d}g$$

abstract Kurzweil-Stieltjes integral: Banach space X

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Application to functional differential equations

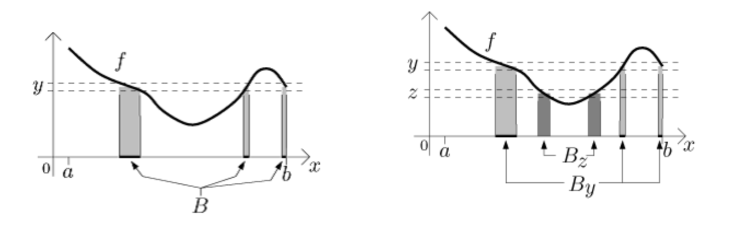
$$\dot{y}(t) = f(y_t, t) \qquad [f(\cdot, t) \text{ linear}]$$

$$\updownarrow$$

$$x(t) = x(a) + \int_a^t \mathrm{d}[A(s)]\, x(s), \quad t \in [a, b\,]$$

Dobrakov integral

Lebesgue type, strong integral



Lebesgue:

$$\int_a^b f \,\mathrm{d}m \approx \sum y\, m(B_y) =: \int_a^b \underbrace{\left(\sum y\, \chi_{B_y}\right)}_{\text{simple function}} \,\mathrm{d}m$$

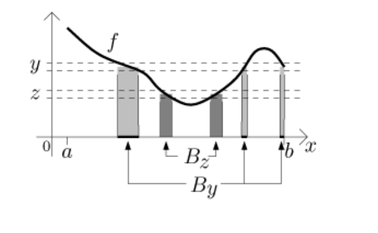
Dobrakov integral

abstract setting: $f : T \to X$

$f_n : T \to X$ simple functions

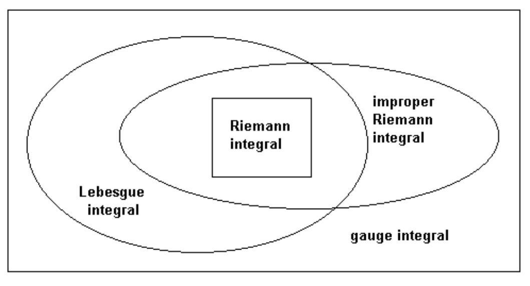
$f_n(t) \to f(t)$ for m-a. e.

$$\int_E f \,\mathrm{d}m = \lim_{n\to\infty} \int_E f_n \mathrm{d}m$$



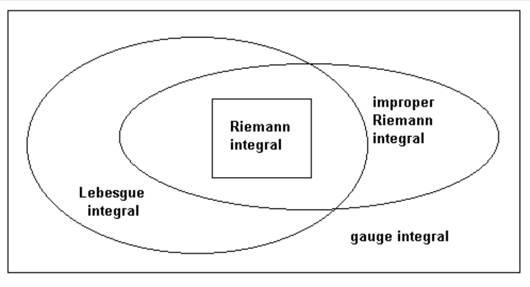
"The class of integrable functions is the smallest class of functions containing the class of simple integrable functions for which the fundamental theorem on interchange of limit and integral is valid."(I. Dobrakov, 1970)

The project:



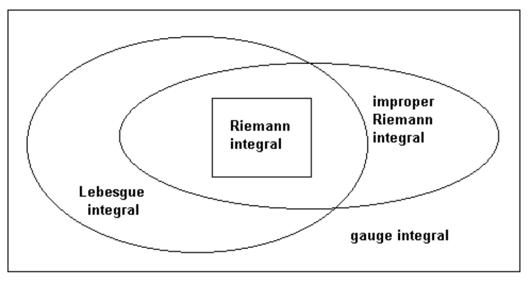
Riemann
integral
improper
Riemann
integral
Lebesgue
integral
gauge integral

The project:



Kurzweil-Stieltjes integral

The project:



Kurzweil-Stieltjes integral

G.A. Monteiro, U.M. Hanung and M. Tvrdý: Bounded convergence theorem for abstract Kurzweil-Stieltjes integral, Monatshefte für Mathematik (2015)

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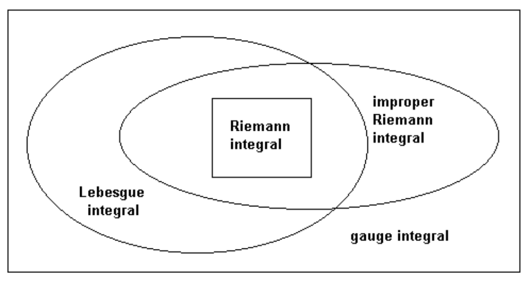

Dobrakov integral??

Kurzweil-Stieltjes integral

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Objectives:

- continuation of the research on abstract Kurzweil-Stieltjes integral over sets
  - investigate convergence results
  - analyse the effects of the nature of the codomain of integrable functions.
  - obtain condition for integration over more general sets.
- connection with the Dobrakov integral
  - study of Dobrakov's theory of integration.
  - comparative analysis of Dobrakov's and Kurzweil's approaches.

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Future Implications:

- a possible new approach for dealing with functional differential equations
- continuation of the tradition of the MI SAS in this research field

Thank you!

Děkuji! Ďakujem!