Networks of Evolutionary Processors with Resources Restricted Filters

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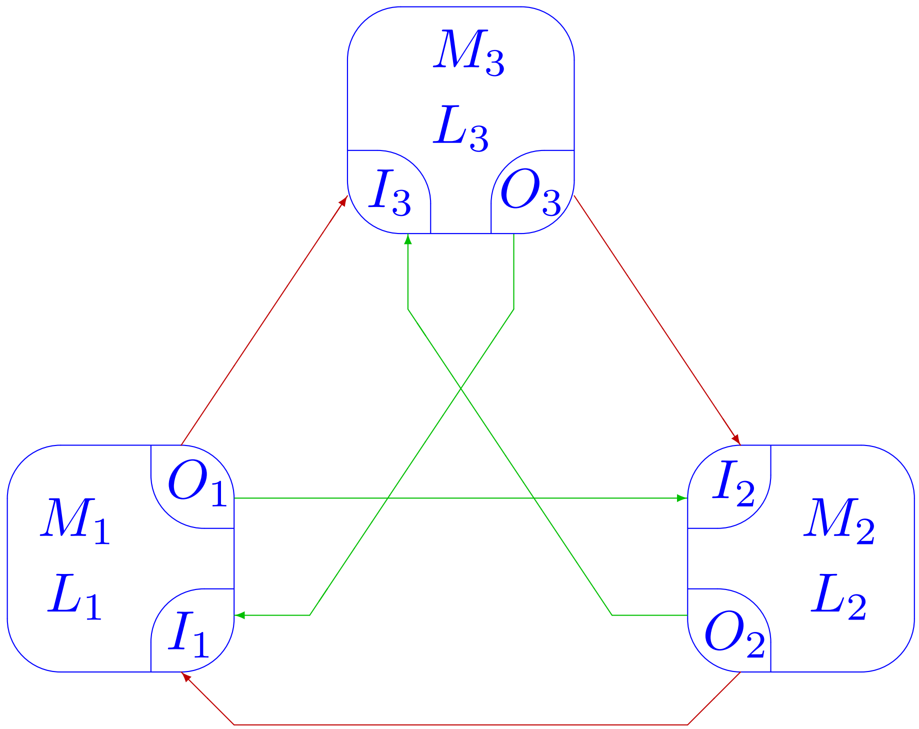
Introduction



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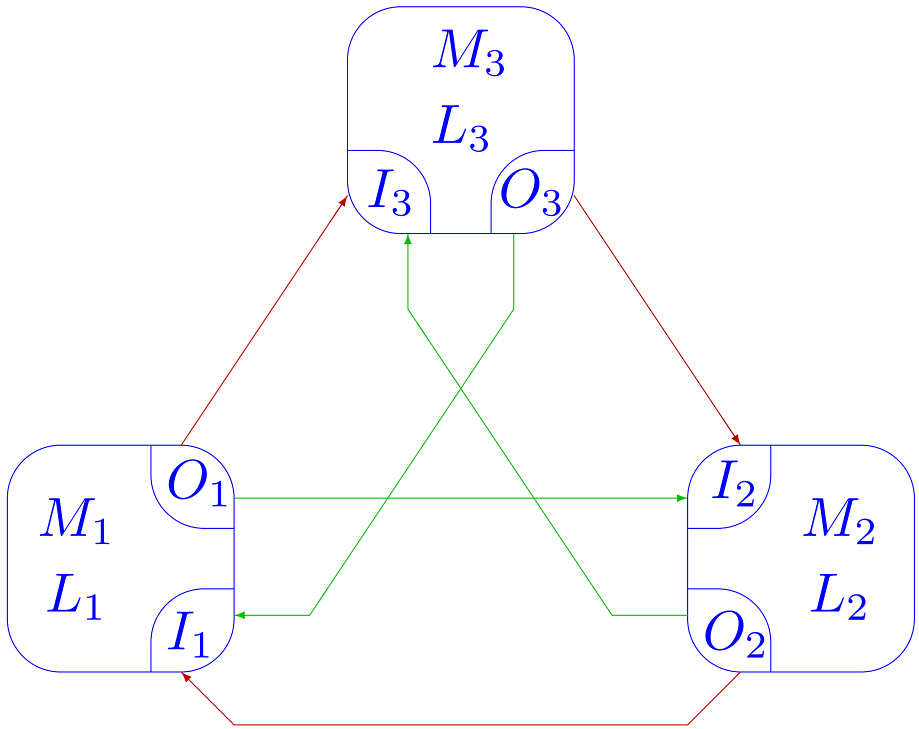

[3] E. Csuhaj-Varjú, A. Salomaa: In *New Trends in Formal Languages*, 1997
[1] J. Castellanos, C. Martín-Vide, V. Mitrana, J. Sempere: In *LNCS 2084*, 2001

Definitions

NEP: $\mathcal{N} = (V, N_1, N_2, \ldots, N_n, E, j)$

Processor: $N_i = (M_i, A_i, I_i, O_i)$

substituting: $M_i \subseteq \{ a \to b \mid a, b \in V \}$
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Language generated: $L(\mathcal{N}) = \bigcup_{t \geq 0} L_j(t)$

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[2] J. Castellanos, C. Martín-Vide, V. Mitrana, J. M. Sempere: Networks of Evolutionary Processors (2003)

[4] J. Dassow, F. Manea, BT: Networks of Evolutionary Processors: The Power of Subregular Filters (2013)

New Results

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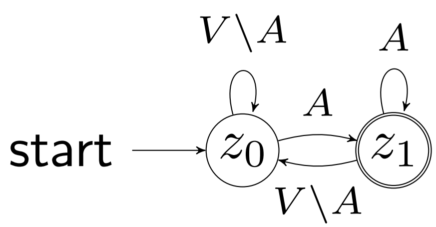

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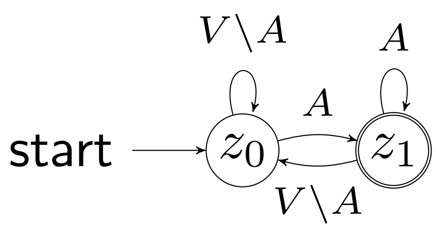

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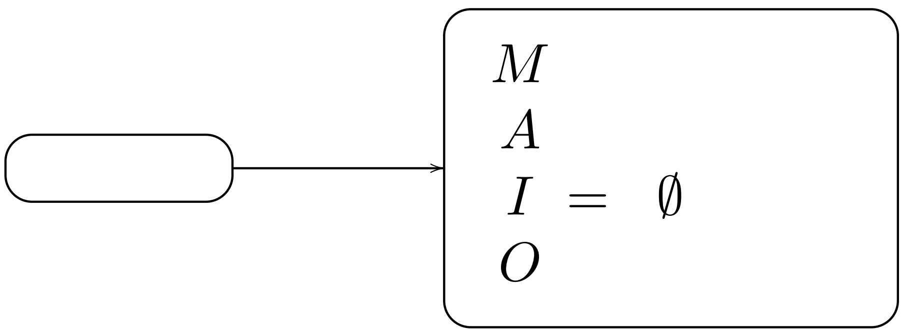

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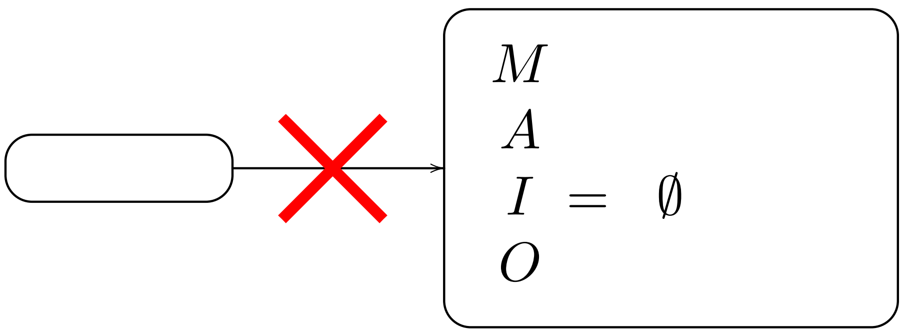

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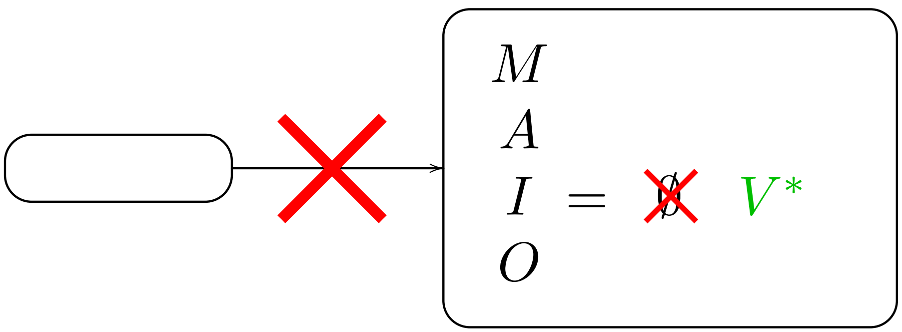

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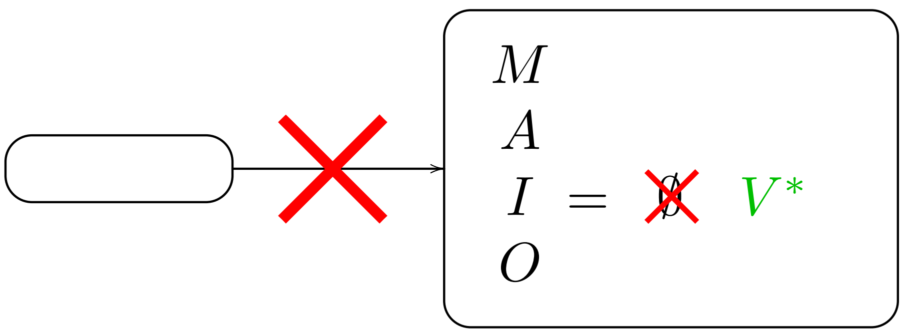

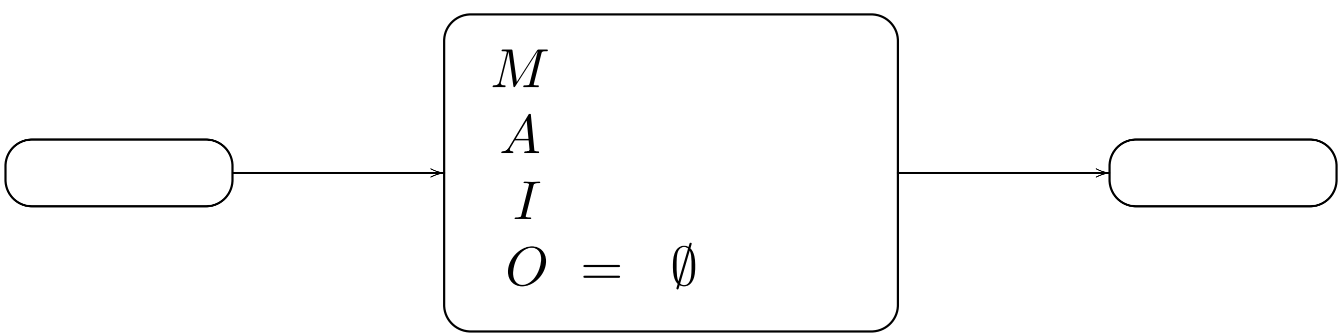

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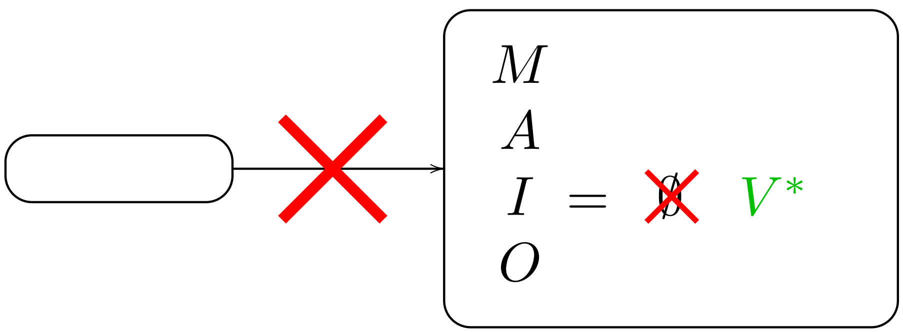

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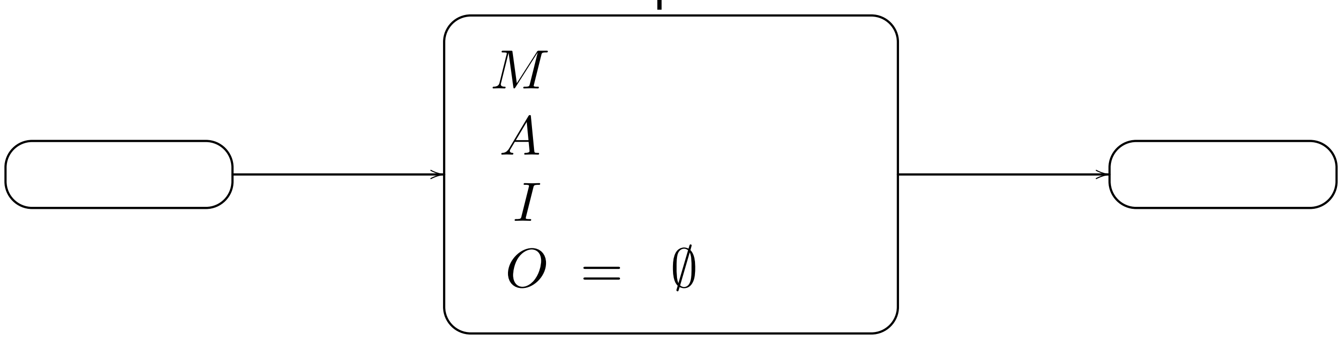

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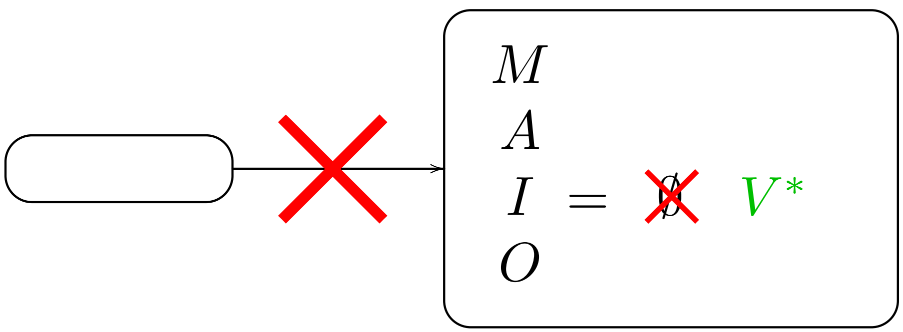

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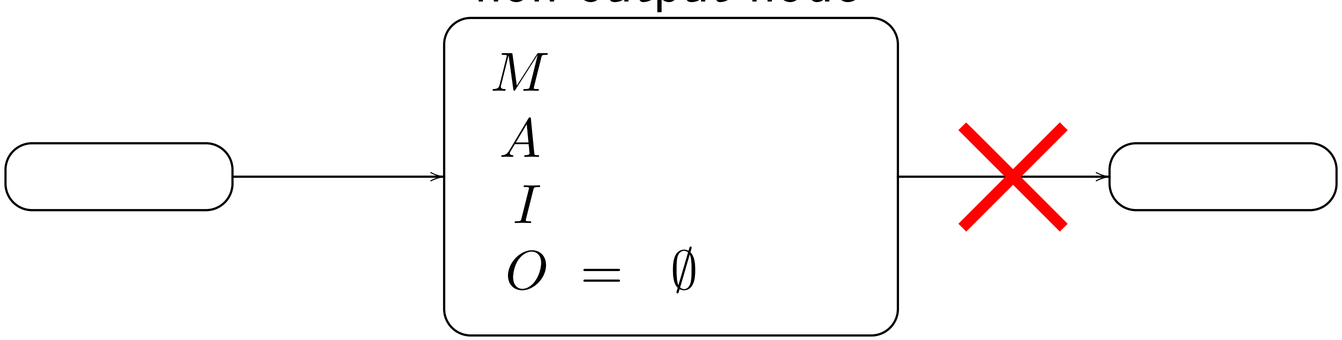

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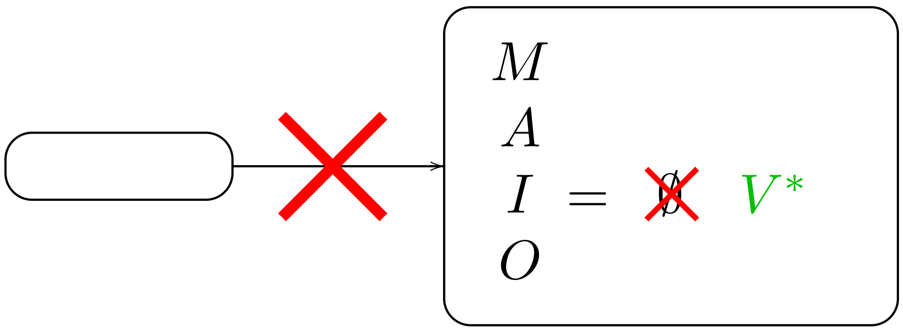

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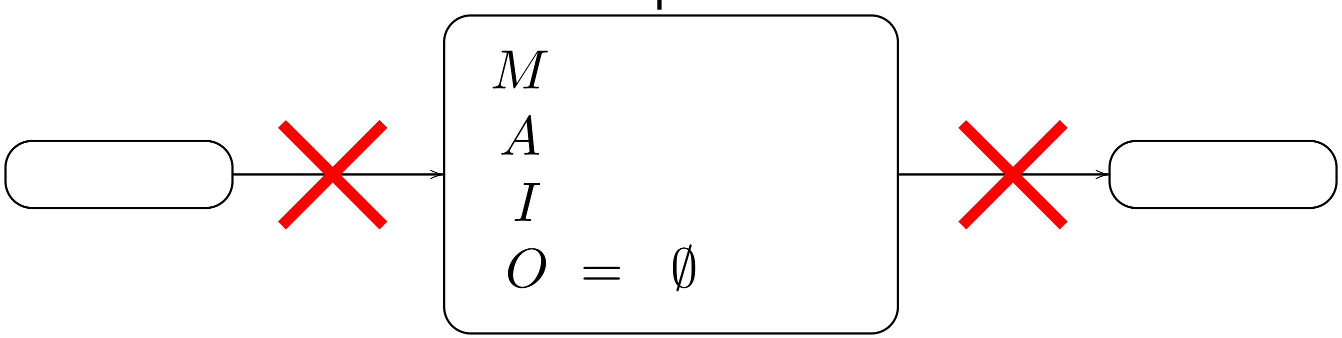

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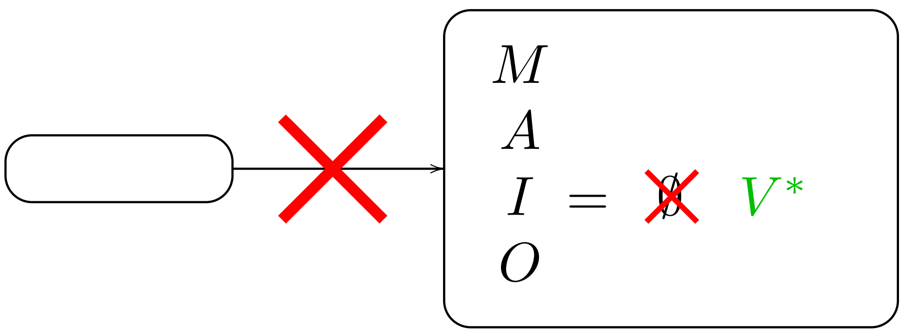

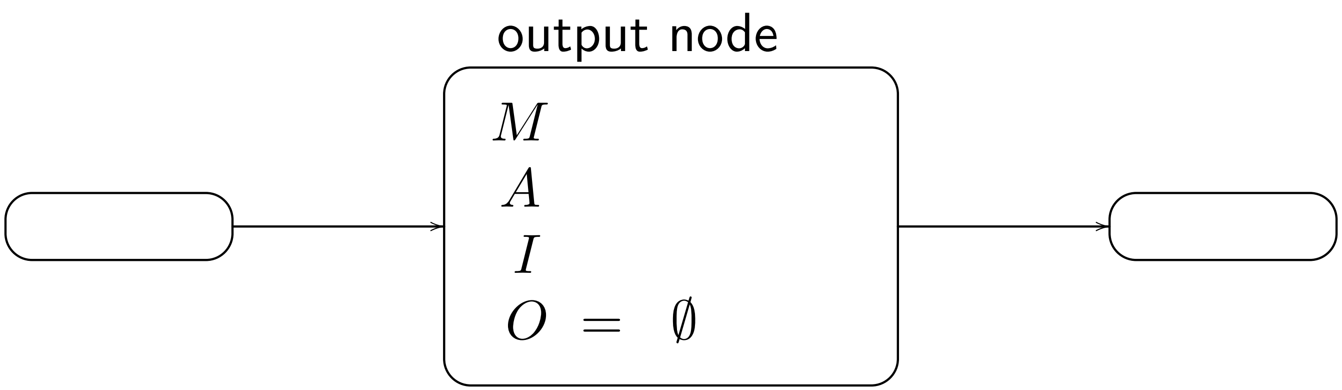

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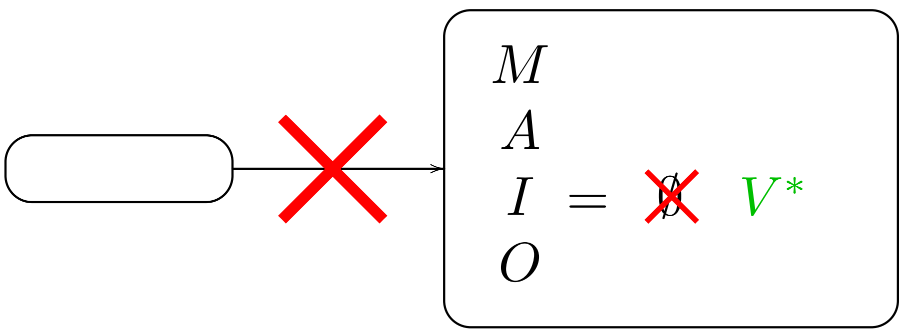

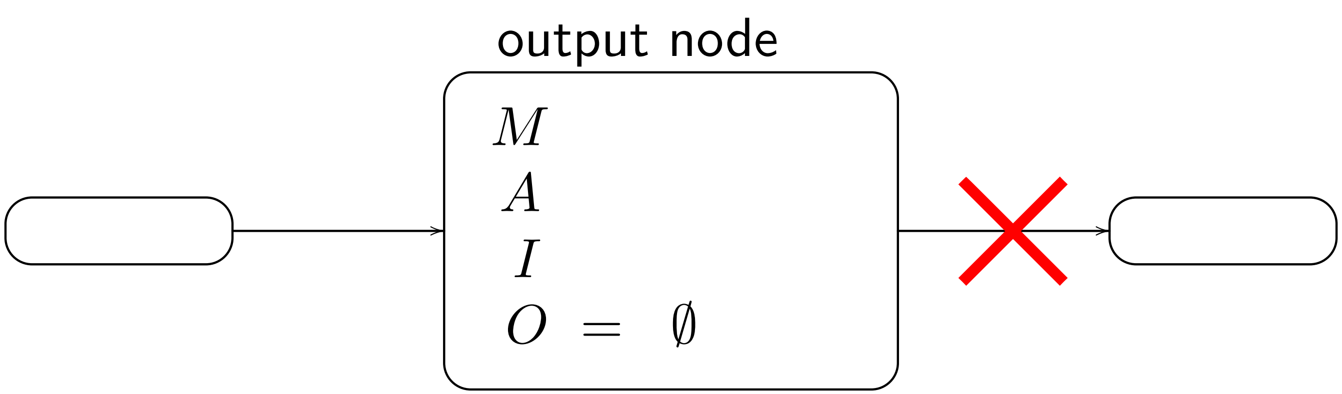

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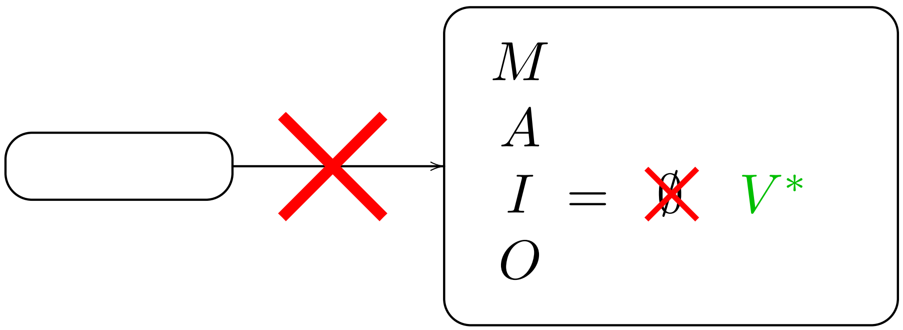

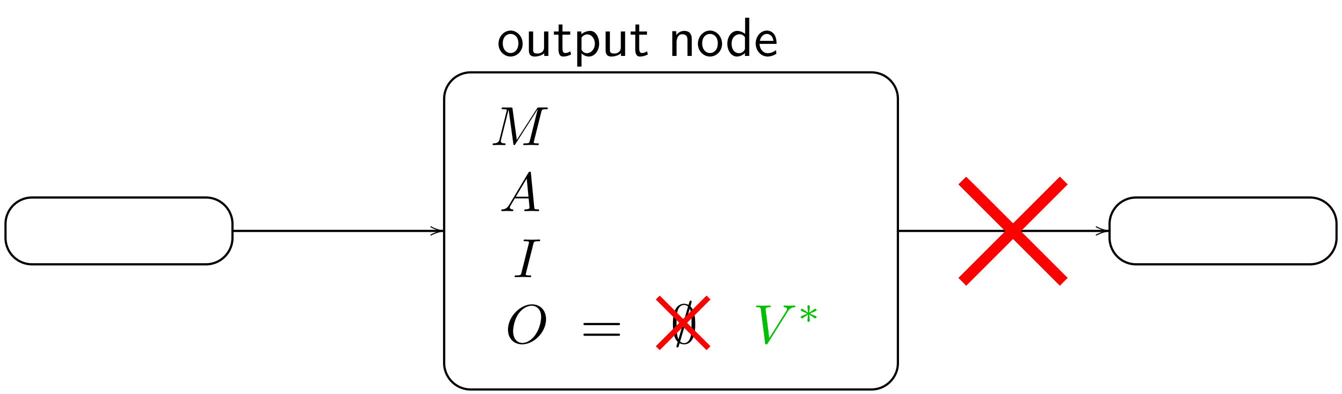

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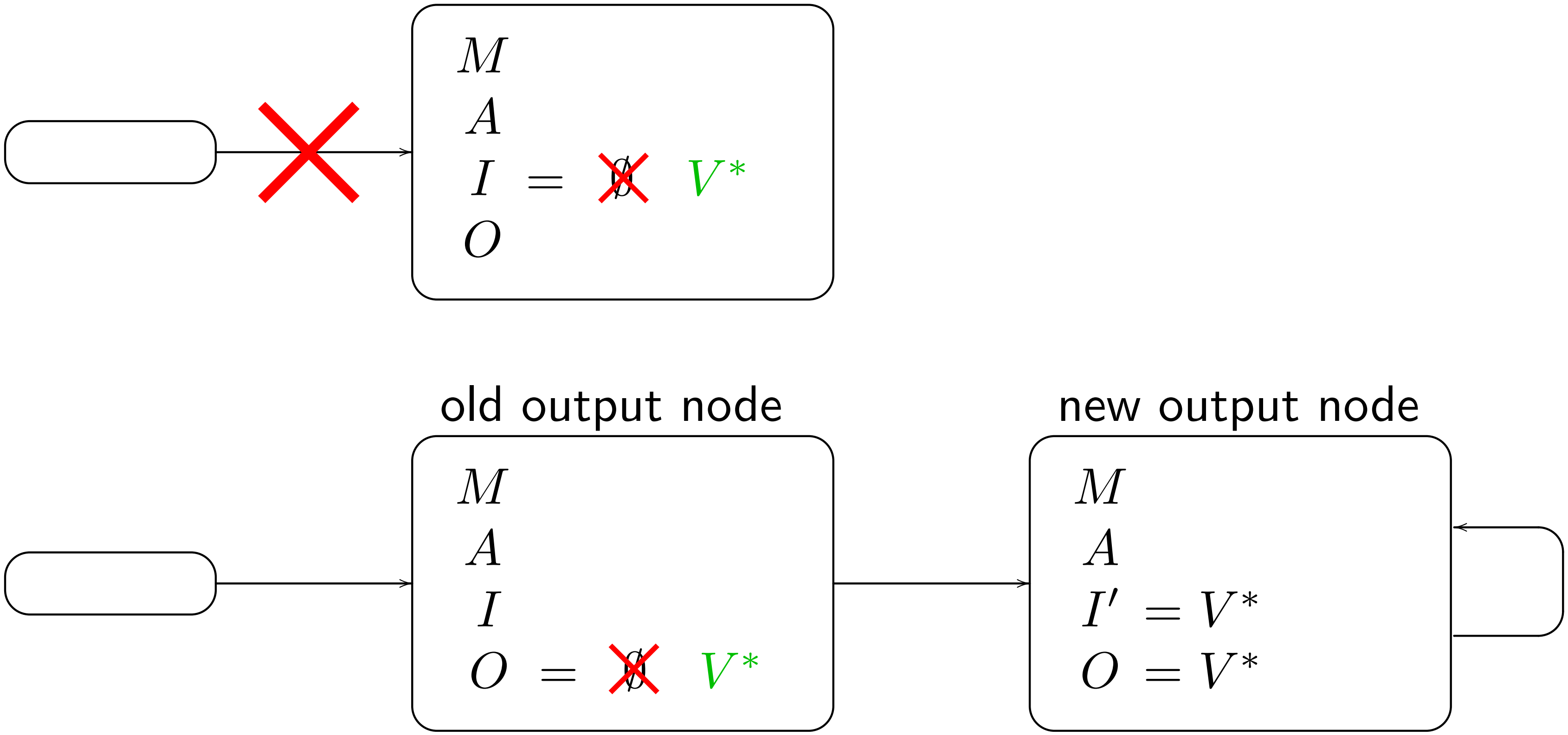

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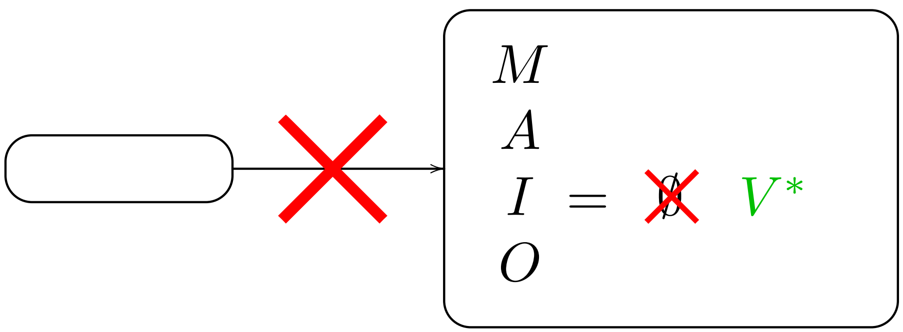

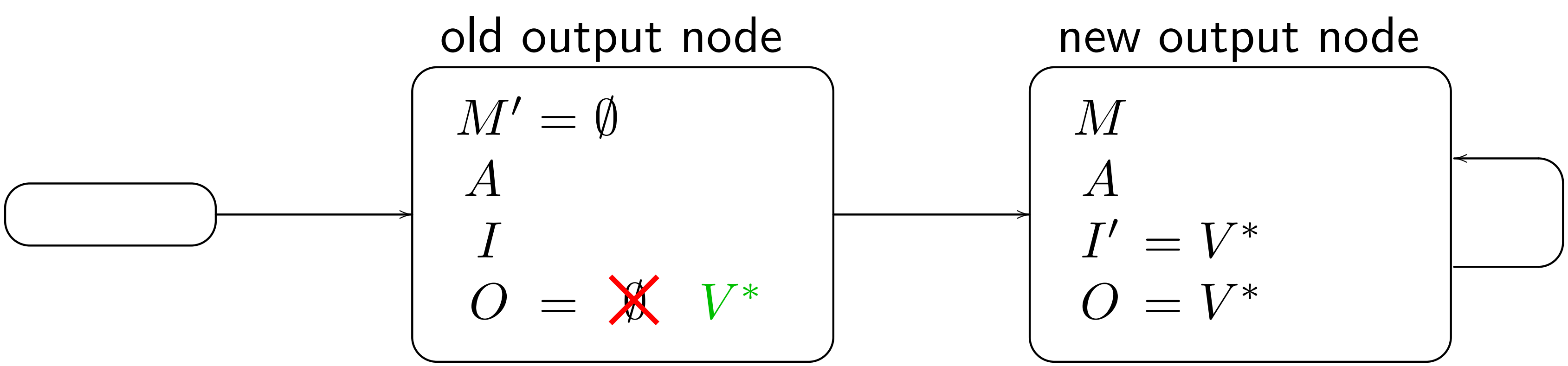

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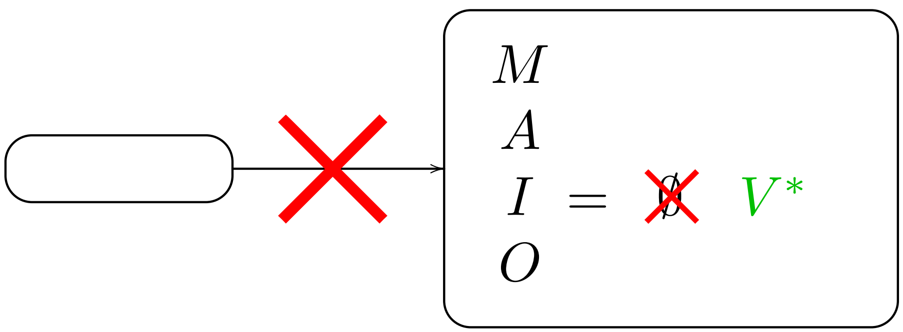

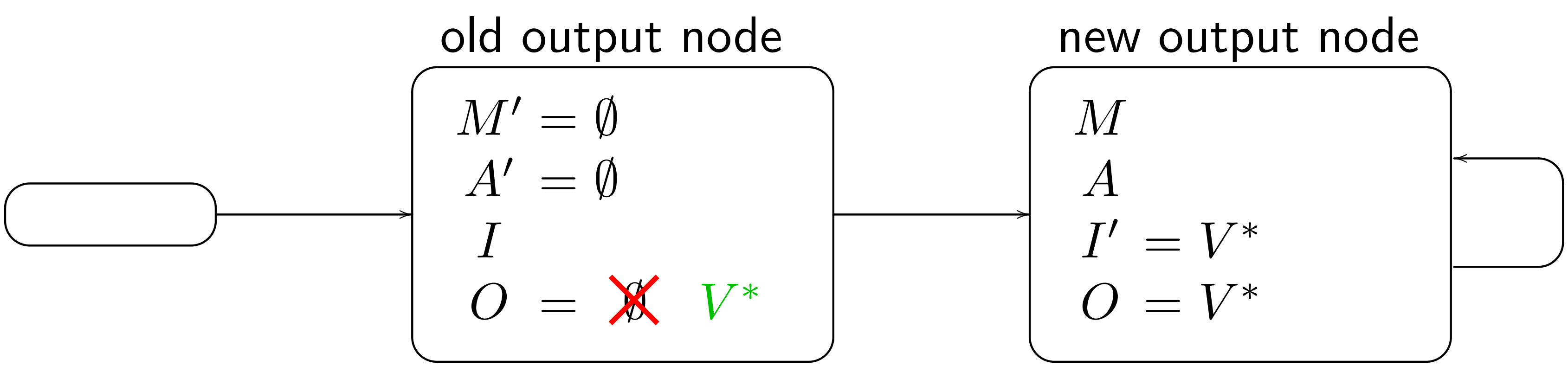

Proofs

$$RE = \mathcal{E}(REG) = \mathcal{E}(PS) = \mathcal{E}(NC) = \mathcal{E}(ORD) = \mathcal{E}(UF)$$
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$$\mathcal{E}(NIL) = \mathcal{E}(COMM) = \mathcal{E}(MON) = \mathcal{E}(REG_1^Z)$$

$$\cdots$$

$$\mathcal{E}(FIN)$$

$$\mathcal{E}(RL_2^P)$$

$$\mathcal{E}(RL_1^P)$$

$$FIN$$

Proofs

$FIN \subset \mathcal{E}(RL_1^P)$

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$\mathcal{E}(RL_1^P) \subset \mathcal{E}(FIN)$

Proofs

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$\mathcal{E}(RL_1^P) \subset \mathcal{E}(FIN)$:

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  $\lambda \to a$ is in the output node

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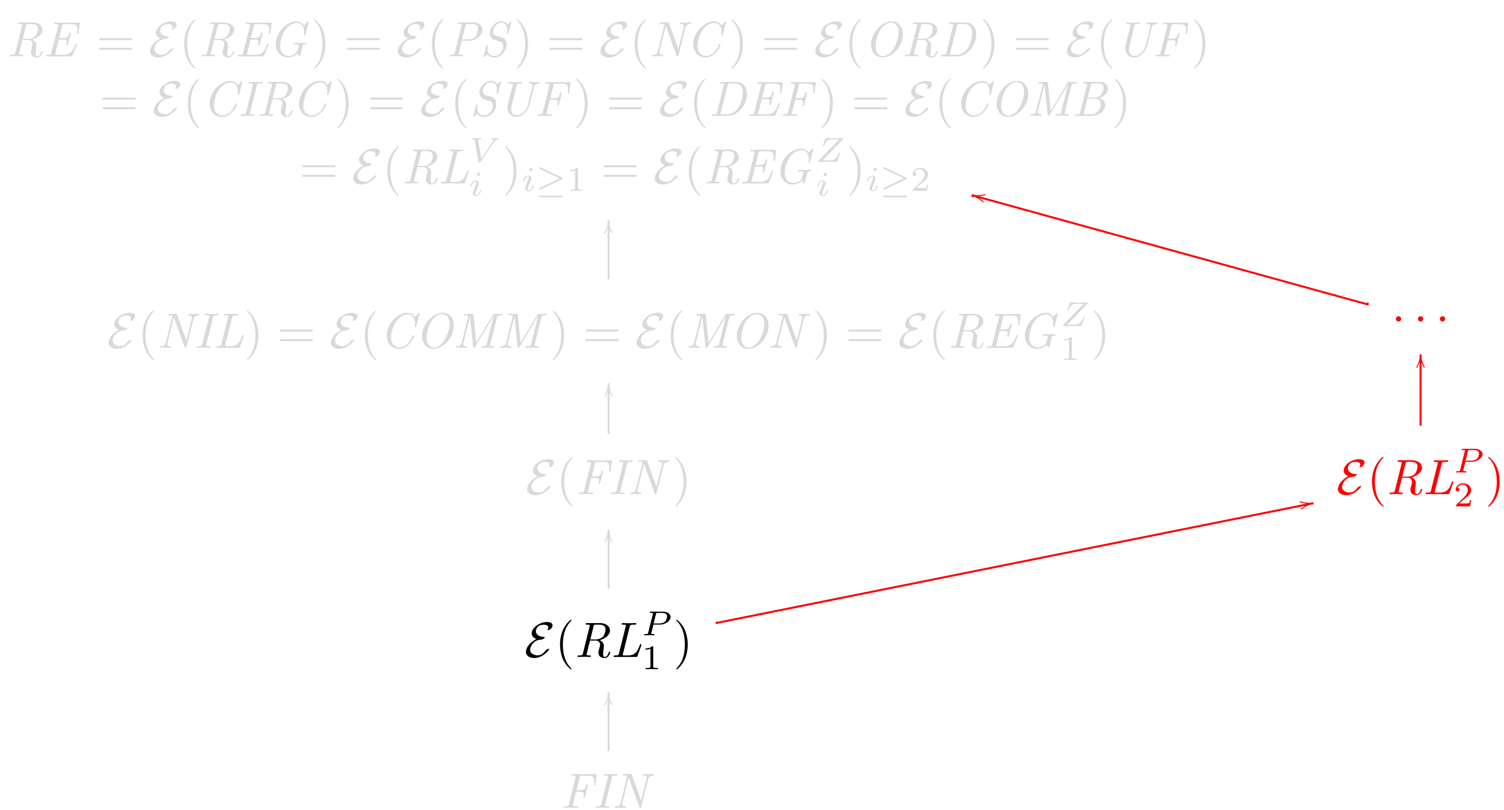
  $\lambda \to a$ is in the output node $\quad \rightsquigarrow \quad$ a^4 or a^7 are also generated

Proofs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$\mathcal{E}(RL_2^P)$
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FIN

Proofs

$\forall i \geq 1: \ \mathcal{E}(RL_i^P) \subset \mathcal{E}(RL_{i+1}^P)$

Proofs

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Let $V_i = \{a_1, \ldots, a_i\}$, $C_i^{(\ell)} = \{ \ a_j^\ell \mid 1 \leq j \leq i \ \}$, and $D_i^{(\ell)} = C_i^{(\ell-1)} \sqcup\!\sqcup \, V_i$. Consider as witness languages

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n.$$

Proofs

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Idea as before:

$$L_2 = \{a^2, a^3, a^5, a^6\} \cup \{ \ a^n \mid n \geq 8 \ \}$$

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Idea as before:

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$$L_3 = (\{a_1^2, a_2^2\} \cup D_2^{(3)}) \cup (\{a_1^5, a_2^5\} \cup D_2^{(6)}) \cup (\{a_1^8, a_2^8\} \cup D_2^{(9)}) \cup V_2^{\geq 11}$$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \in \mathcal{E}(RL_{i+1}^P)$$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \in \mathcal{E}(RL_{i+1}^P):$$

$$M_1 = \{ \lambda \to x \mid x \in V_i \},$$
$$A_1 = V_i^{3(i+1)+1},$$
$$I_1 = V_i^*,$$
$$O_1 = V_i^*$$

output node

$$M_2 = \emptyset,$$
$$A_2 = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}),$$
$$I_2 = V_i^*,$$
$$O_2 = V_i^*$$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \in \mathcal{E}(RL_{i+1}^P):$$

output node

$$M_1 = \{ \lambda \to x \mid x \in V_i \},\quad A_1 = V_i^{3(i+1)+1},\quad I_1 = V_i^*,\quad O_1 = V_i^*$$

$$M_2 = \emptyset,\quad A_2 = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}),\quad I_2 = V_i^*,\quad O_2 = V_i^*$$

V_i^* is generated by the $i+1$ right-linear rules $S \to a_1 S, \ldots, S \to a_i S, S \to \lambda$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$.

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

- ... no rules

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

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Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

- ... no rules $\rightsquigarrow I_o = L_{i+1} \setminus A_o \rightsquigarrow i$ production rules not sufficient ↯

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

- ... no rules $\leadsto I_o = L_{i+1} \setminus A_o \leadsto i$ production rules not sufficient ↯
- ... insertion rules

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

- ... no rules $\rightsquigarrow I_{\mathrm{o}} = L_{i+1} \setminus A_{\mathrm{o}} \rightsquigarrow i$ production rules not sufficient ↯
- ... insertion rules $\rightsquigarrow O_{\mathrm{o}} \supseteq \bigcup_{k=1}^{i+1} D_i^{(3k)}$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

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- ... deletion rules

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

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- ... deletion rules $\rightsquigarrow O_\mathrm{o} \supseteq \bigcup_{k=1}^{i+1} V_i^{3k+2} \cap L_{i+1}$

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

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Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

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- ... substitution rules

Proofs

$$L_{i+1} = \bigcup_{k=1}^{i+1} (C_i^{(3k-1)} \cup D_i^{(3k)}) \cup \bigcup_{n \geq 3(i+1)+2} V_i^n \notin \mathcal{E}(RL_i^P):$$

Assume, $L_{i+1} \in \mathcal{E}(RL_i^P)$. Output node has ...

- ... no rules $\rightsquigarrow I_{\mathrm{o}} = L_{i+1} \setminus A_{\mathrm{o}} \rightsquigarrow i$ production rules not sufficient ↯
- ... insertion rules $\rightsquigarrow O_{\mathrm{o}} \supseteq \bigcup_{k=1}^{i+1} D_i^{(3k)} \rightsquigarrow i$ rules not sufficient ↯
- ... deletion rules $\rightsquigarrow O_{\mathrm{o}} \supseteq \bigcup_{k=1}^{i+1} V_i^{3k+2} \cap L_{i+1} \rightsquigarrow i$ rules not sufficient ↯
- ... substitution rules $\rightsquigarrow$ wrong short words are generated ↯

Hierarchy

$$RE = \mathcal{E}(REG) = \mathcal{E}(PS) = \mathcal{E}(NC) = \mathcal{E}(ORD) = \mathcal{E}(UF)$$

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$$\dots$$

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$$\mathcal{E}(RL_2^P)$$

$$\mathcal{E}(RL_1^P)$$

$$FIN$$

Incomparability Results

$$\forall i \geq 1 : L_{i+1} \in \mathcal{E}(FIN) \setminus \mathcal{E}(RL_i^P)$$

Incomparability Results

$\forall i \geq 1 : L_{i+1} \in \mathcal{E}(FIN) \setminus \mathcal{E}(RL_i^P)$:

$L_{i+1} \notin \mathcal{E}(RL_i^P)$: already proven

Incomparability Results

$\forall i \geq 1 : L_{i+1} \in \mathcal{E}(FIN) \setminus \mathcal{E}(RL_i^P)$:

$L_{i+1} \notin \mathcal{E}(RL_i^P)$: already proven

$L_{i+1} \in \mathcal{E}(FIN)$:

output node

$$M_1 = \{\ \lambda \to x \mid x \in V_i\ \},\quad A_1 = V_i^{3(i+1)+2},\quad I_1 = A_2,\quad O_1 = \bigcup_{k=1}^{i+1} D_i^{(3k)}$$

$$M_2 = \emptyset,\quad A_2 = \bigcup_{k=1}^{i+1} C_i^{(3k-1)},\quad I_2 = \emptyset,\quad O_2 = A_2$$

Incomparability Results

$$L = \{a\}^*\{b\} \in \mathcal{E}(RL_2^P) \setminus \mathcal{E}(MON)$$

Incomparability Results

$L = \{a\}^*\{b\} \in \mathcal{E}(RL_2^P) \setminus \mathcal{E}(MON)$:

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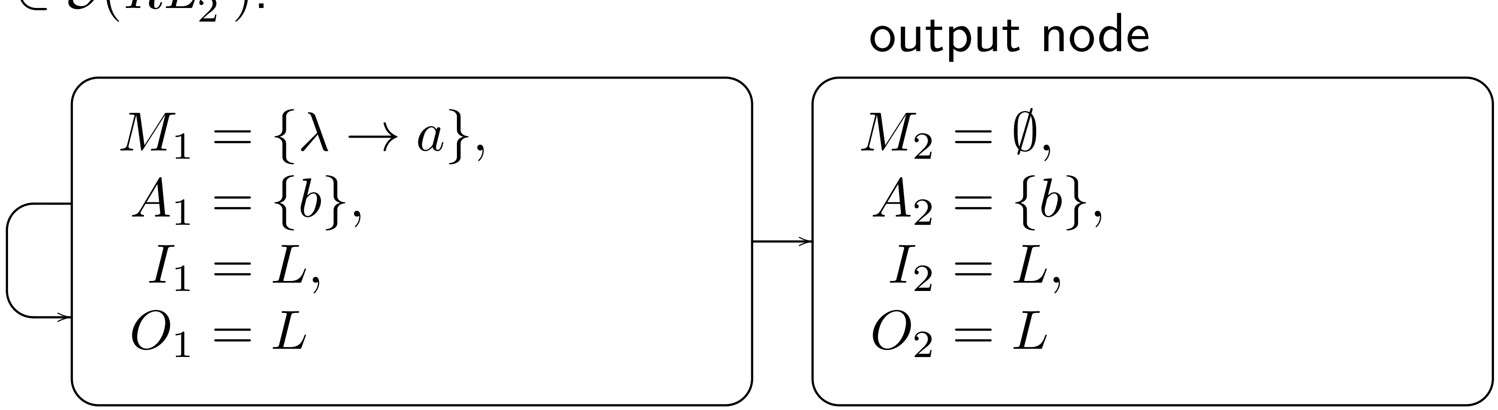


Incomparability Results

$L = \{a\}^*\{b\} \in \mathcal{E}(RL_2^P) \setminus \mathcal{E}(MON)$:

$L \in \mathcal{E}(RL_2^P)$:

output node

$M_1 = \{\lambda \to a\}$,
$A_1 = \{b\}$,
$I_1 = L$,
$O_1 = L$

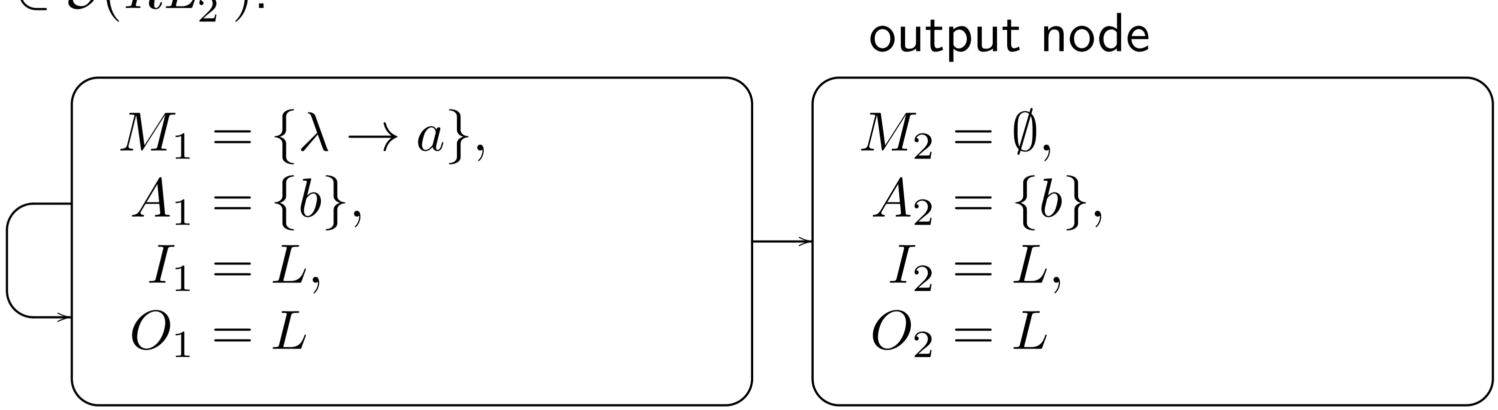
$M_2 = \emptyset$,
$A_2 = \{b\}$,
$I_2 = L$,
$O_2 = L$

Filter $L = \{a\}^*\{b\}$ is generated by the rules $S \to aS$ and $S \to b$.

Incomparability Results

$L = \{a\}^*\{b\} \in \mathcal{E}(RL_2^P) \setminus \mathcal{E}(MON)$:

$L \in \mathcal{E}(RL_2^P)$:



Filter $L = \{a\}^*\{b\}$ is generated by the rules $S \to aS$ and $S \to b$.

$L \notin \mathcal{E}(MON)$:

Letter a is inserted infinitely often, a can appear after b
$\rightsquigarrow$ those words cannot be filtered out by monoidal filters

Summary

$$RE = \mathcal{E}(REG) = \mathcal{E}(PS) = \mathcal{E}(NC) = \mathcal{E}(ORD) = \mathcal{E}(UF)$$
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Future Research

Investigation of . . .

Future Research

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- ... the relations to other families, e. g., to those considered in [6]

[6] J. Dassow, BT: Networks with evolutionary processors and ideals and codes as filters (2018)

Future Research

Investigation of ...

- ... the relations to other families, e. g., to those considered in [6]
- ... accepting networks of evolutionary processors with resources restricted filters and comparision of the computational power with respect to those considered in [9]

[6] J. Dassow, BT: Networks with evolutionary processors and ideals and codes as filters (2018)

[9] F. Manea, BT: Accepting networks of evolutionary processors with subregular filters 2014