

THE EGOROFF THEOREM FOR OPERATOR-VALUED MEASURES IN LOCALLY CONVEX SPACES

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Abstract. The Egoroff theorem for measurable \mathbf{X} -valued functions and operator-valued measures $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$, where Σ is a σ -algebra of subsets of $T \neq \emptyset$ and \mathbf{X}, \mathbf{Y} are both locally convex spaces, is proved. The measure is supposed to be atomic and the convergence of functions is net.

1 Introduction

The classical Egoroff theorem states that the almost everywhere convergence implies the almost uniform convergence on a finite measure space, cf. [5]. A necessary and sufficient condition for a sequence of measurable real functions to be almost uniformly convergent is given in [2]. In generalizing to functions taking values in more general spaces, some problems appear arising from the fact that the classical relationship between the pointwise convergence and the convergence in measure is not saved. Namely, any convergence almost everywhere does not imply convergence in measure in general. For a complete measure space (T, Σ, μ) , and a locally convex space \mathbf{X} the following condition may be considered, cf. [4]:

Let \mathcal{M} be a family of \mathbf{X} -valued functions defined on T . The locally convex space \mathbf{X} is said to satisfy the finite Egoroff condition with respect to \mathcal{M} if and only if every sequence in \mathcal{M} which converges almost everywhere to a function, is almost uniformly convergent on every measurable set $A \in \Sigma$ of a finite measure.

However, it is not so easy to find such a family of functions \mathcal{M} . Thus, E. Wagner and W. Wilczyński proved the following theorem in [13].

Theorem 1.1 *If a measurable space (T, Σ, μ) fulfils the countable chain condition, then the convergence I -a.e. is equivalent to the convergence with respect to the σ -ideal I if and only if Σ/I is atomic.*

In the operator-valued measure theory in Banach spaces the pointwise convergence (of sequences) of measurable functions on a set of finite semivariation implies the convergence in (continuous) semivariation of the measure $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$, where Σ is a σ -algebra of subsets of a set $T \neq \emptyset$, and \mathbf{X}, \mathbf{Y} are Banach spaces, cf. [1], and [3]. If \mathbf{X} fails to be metrizable, the relationship between these two convergences is quite unlike the classical situation, cf. Example after Definition 1.11 in [12].

It is also well-known that the Egoroff theorem cannot hold for arbitrary nets of measurable functions without some restrictions on measure, net convergence

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of functions, or class of measurable functions. In [6], Definition 1.2, the first author introduced the so called Condition (GB) under which everywhere convergence of net of measurable functions implies convergence of these functions in semivariation on a set of finite variation of measure in locally convex setting, cf. [7], Theorem 3.3 (without the assumption of countable chain condition). This condition concerns families of submeasures and enables to work with nets of measurable functions instead of sequences. Also, Condition (GB) is a generalization of Condition (B) which is important for sequences in the classical measure and integration theory, cf. [11]. The analogous condition for nets in the classical setting was introduced and investigated by B. F. Gogvadze, cf. [4].

Recall that Condition (GB) is fulfilled in the case of atomic operator-valued measures, cf. [7]. Atomic measures are not of a great interest in the classical theory of measure and integral, because they lead only to considerations of absolutely convergent series. But when we consider measures with very general range space, e.g. a locally convex space, the situation changes.

In this paper we prove the Egoroff theorem for atomic operator-valued measures in locally convex topological vector spaces.

2 Preliminaries

The description of the theory of locally convex topological vector spaces may be found in [9]. By a *net* (with values in a set S) we mean a function from I to S , where I is a directed partially ordered set. Throughout this paper I is a directed index set representing direction of a net. For terminology concerning nets, see [10].

Let T be a non-void set and let Σ be a σ -algebra of subsets of T . By 2^T we denote the potential set of T . Let \mathbf{X} and \mathbf{Y} be two Hausdorff locally convex topological vector spaces over the field \mathbb{K} of all real \mathbb{R} or complex numbers \mathbb{C} , with two families of seminorms P and Q defining the topologies on \mathbf{X} and \mathbf{Y} , respectively. Let $L(\mathbf{X}, \mathbf{Y})$ denote the space of all continuous linear operators $L : \mathbf{X} \rightarrow \mathbf{Y}$, and let \mathbb{N} be the set of all natural numbers.

In what follows $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ is an operator-valued measure σ -additive in the strong operator topology of the space $L(\mathbf{X}, \mathbf{Y})$, i.e. $\mathbf{m}(\cdot)\mathbf{x} : \Sigma \rightarrow \mathbf{Y}$ is a \mathbf{Y} -valued vector measure for every $\mathbf{x} \in \mathbf{X}$.

Definition 2.1 Let $p \in P$, $q \in Q$, $E \in \Sigma$ and $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$.

- (a) The (p, q) -semivariation of the measure \mathbf{m} is the set function $\hat{\mathbf{m}}_{p,q} : \Sigma \rightarrow [0, \infty]$, defined as follows:

$$\hat{\mathbf{m}}_{p,q}(E) = \sup q \left(\sum_{n=1}^N \mathbf{m}(E_n \cap E)\mathbf{x}_n \right),$$

where the supremum is taken over all finite disjoint partitions $\{E_n \in \Sigma; E \subset \bigcup_{n=1}^N E_n, E_n \cap E_m = \emptyset, n \neq m, m, n = 1, 2, \dots, N\}$ of E , and all finite sets $\{\mathbf{x}_n \in \mathbf{X}; p(\mathbf{x}_n) \leq 1, n = 1, 2, \dots, N\}$, $N \in \mathbb{N}$.

- (b) The (p, q) -variation of the measure \mathbf{m} is the set function $\mathbf{var}_{p,q}(\mathbf{m}, \cdot) :$

$\Sigma \rightarrow [0, \infty]$, defined by the equality

$$\mathbf{var}_{p,q}(\mathbf{m}, E) = \sup \sum_{n=1}^N q_p(\mathbf{m}(E_n \cap E)),$$

where the supremum is taken over all finite disjoint partitions $\{E_n \in \Sigma; E = \bigcup_{n=1}^N E_n, E_n \cap E_m = \emptyset, n \neq m, n, m = 1, 2, \dots, N, N \in \mathbb{N}\}$ of E and

$$q_p(\mathbf{m}(F)) = \sup_{p(\mathbf{x}) \leq 1} q(\mathbf{m}(F)\mathbf{x}), \quad F \in \Sigma.$$

- (c) The *inner* (p, q) -semivariation of the measure \mathbf{m} is the set function $\hat{\mathbf{m}}_{p,q}^* : 2^T \rightarrow [0, \infty]$, defined as follows:

$$\hat{\mathbf{m}}_{p,q}^*(E) = \sup_{F \subset E, F \in \Sigma} \hat{\mathbf{m}}_{p,q}(F), \quad E \in 2^T.$$

Note that $\hat{\mathbf{m}}_{p,q}(E) \leq \mathbf{var}_{p,q}(\mathbf{m}, E)$ for every $q \in Q, p \in P$, and $E \in \Sigma$. The following lemma is obvious.

Lemma 2.2 *For every $p \in P$ and $q \in Q$, the (p, q) -(semi)variation of the measure \mathbf{m} is a monotone and σ -additive (σ -subadditive) set function, and $\mathbf{var}_{p,q}(\mathbf{m}, \emptyset) = 0, (\hat{\mathbf{m}}_{p,q}(\emptyset) = 0)$.*

Definition 2.3 Let $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$.

- (a) The set $E \in \Sigma$ is said to be of *positive* (p, q) -variation of the measure \mathbf{m} if there exist $q \in Q, p \in P$, such that $\mathbf{var}_{p,q}(\mathbf{m}, E) > 0$.
- (b) We will say that $E \in \Sigma$ is a *$\hat{\mathbf{m}}$ -null set* if $\hat{\mathbf{m}}_{p,q}(E) = 0$ for every $q \in Q, p \in P$.
- (c) We say that the set $E \in \Sigma$ is of *finite* (p, q) -variation of the measure \mathbf{m} if to every $q \in Q$ there exists $p \in P$, such that $\mathbf{var}_{p,q}(\mathbf{m}, E) < \infty$. We will denote this relation shortly $Q \rightarrow_E P$, or, $q \mapsto_E p$, for $q \in Q, p \in P$.

Remark 2.4 The relation $Q \rightarrow_E P$ may be different for different sets $E \in \Sigma$ of finite (p, q) -variation of the measure \mathbf{m} .

Definition 2.5 A measure $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ is said to *satisfy Condition (GB)* if for every $E \in \Sigma$ of finite positive (p, q) -variation and every net of sets $E_i \in \Sigma, E_i \subset E, i \in I$, there holds

$$\limsup_{i \in I} E_i \neq \emptyset,$$

whenever there exist real numbers $\delta(q, p, E) > 0, p \in P, q \in Q$ such that $\hat{\mathbf{m}}_{p,q}(E_i) \geq \delta(q, p, E)$ for every $i \in I$ and every couple $(p, q) \in P \times Q$, with $q \mapsto_E p$.

Note that the everywhere convergence of a net of measurable functions implies the convergence of these functions in (p, q) -semivariation on a set of finite (p, q) -variation if and only if Condition (GB) is fulfilled. For instance, the Lebesgue measure does not satisfy Condition (GB) on the real line (for arbitrary nets of sets).

Definition 2.6 We say that a set $E \in \Sigma$ of positive (p, q) -semivariation of the measure \mathbf{m} is an $\hat{\mathbf{m}}$ -atom if every proper subset A of E is either \emptyset or $A \notin \Sigma$.

We say that the measure \mathbf{m} is *atomic* if each $E \in \Sigma$ can be expressed in the form $E = \bigcup_{k=1}^{\infty} A_k$, where $A_k, k \in \mathbb{N}$, are $\hat{\mathbf{m}}$ -atoms.

In [7] it is shown that a class of measures satisfying Condition (GB) is non-empty and the following result is proved.

Theorem 2.7 *If \mathbf{m} is a (countable) purely atomic operator-valued measure, then \mathbf{m} satisfies Condition (GB).*

Definition 2.8 We say that a function $\mathbf{f} : T \rightarrow \mathbf{X}$ is *measurable* if $\{t \in T; p(\mathbf{f}(t)) \geq \eta\} \in \Sigma$ for every $\eta > 0$ and $p \in P$.

The following result is well-known, cf. [7].

Lemma 2.9 *If there exists a non-measurable set E , such that $E \subset E_0$, $E_0 \in \Sigma$, and every finite subset of E is measurable, then the set of all measurable functions is not closed with respect to pointwise limits of measurable functions.*

Definition 2.10 A net $\mathbf{f}_i, i \in I$, of measurable functions is said to be \mathbf{m} -almost uniformly convergent to a measurable function \mathbf{f} on $E \in \Sigma$ if for every $\varepsilon > 0$ and every $q \in Q, p \in P$ with $q \rightarrow_E p$ there exist measurable sets $E(\varepsilon, p, q)$, such that

$$\lim_{i \in I} \|\mathbf{f}_i - \mathbf{f}\|_{E \setminus E(\varepsilon, p, q)} = 0 \quad \text{and} \quad \hat{\mathbf{m}}_{p, q}(E(\varepsilon, p, q)) < \varepsilon.$$

In [8] the concept of generalized strong continuity of the (p, q) -semivariation of the measure is introduced. Note that this notion enables development of the concept of an integral with respect to the $L(\mathbf{X}, \mathbf{Y})$ -valued measure based on the net convergence of simple functions. For this purpose the notion of the inner (p, q) -semivariation is used for this generalization. This way we restrict the set of $L(\mathbf{X}, \mathbf{Y})$ -valued measures which can be taken for such type of integration. For instance, every atomic measure is generalized strongly continuous. So, the class of measures with the generalized strongly continuous (p, q) -semivariation is nonempty.

Definition 2.11 We say that the (p, q) -semivariation of the measure $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ is *generalized strongly continuous (GS-continuous, for short)* if for every set of finite (p, q) -variation $E \in \Sigma$ and every monotone net of sets $E_i \subset T, E_i \subset E, i \in I$, the following equality

$$\lim_{i \in I} \hat{\mathbf{m}}_{p, q}^*(E_i) = \hat{\mathbf{m}}_{p, q}^*(\lim_{i \in I} E_i)$$

holds for every couple $(p, q) \in P \times Q$, such that $q \mapsto_E p$.

The following result connecting the notion of GS-continuity and Condition (GB) was proved in [8].

Theorem 2.12 *If the (p, q) -semivariation of a measure $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ is GS-continuous, then the measure \mathbf{m} satisfies Condition (GB).*

3 Egoroff theorem for atomic operator-valued measures in locally convex spaces

Before proving the main result of this paper we state the following useful lemma.

Lemma 3.1 *If $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ is a (countable) purely atomic measure, then its (p, q) -semivariation is GS-continuous.*

Proof. Let $E \in \Sigma$ be a set of finite and positive (p, q) -variation. Let $E_i, i \in I$, be an arbitrary decreasing net of sets. Recall that $E_i \searrow G (\in 2^T)$ if and only if

- (1) $i \leq j \Rightarrow E_i \supset E_j$, and
- (2) $\bigcap_{i \in I} E_i = G$.

It is clear that it is enough to consider the case $G = \emptyset$, because

$$E_i \searrow G \Leftrightarrow G \subset E_i, \quad E_i \setminus G \searrow \emptyset.$$

First, in the case $E_i \in \Sigma, i \in I$, we have

$$\lim_{i \in I} \hat{\mathbf{m}}_{p,q}^*(E_i) = \lim_{i \in I} \hat{\mathbf{m}}_{p,q}(E_i), \quad (1)$$

and since the family of atoms is at most a countable set, there is

$$\lim_{i \in I} E_i = \bigcap_{i \in I} E_i \in \Sigma$$

and, therefore,

$$\hat{\mathbf{m}}_{p,q}^*(\lim_{i \in I} E_i) = \hat{\mathbf{m}}_{p,q}(\lim_{i \in I} E_i) \quad (2)$$

for every $p \in P, q \in Q$ such that $q \rightarrow_E p$.

Now, take an arbitrary set $E \in \Sigma$ of positive and finite (p, q) -variation. Denote by \mathcal{A} the set of all $\hat{\mathbf{m}}$ -atoms, and put $l(i, E) = (\mathcal{A} \cap E) \setminus E_i, i \in I$. Clearly

$$i \leq j, i, j \in I \Rightarrow l(i, E) \subset l(j, E)$$

and there exist atoms $A_n \in \mathcal{A}, n \in \mathbb{N}$, such that $l(i, E) = \{A_1, A_2, \dots, A_n, \dots\}$.

By Lemma 2.2 we have

$$\mathbf{var}_{p,q}(\mathbf{m}, E) = \mathbf{var}_{p,q}(\mathbf{m}, E_i) + \sum_{A_n \in l(i, E)} \mathbf{var}_{p,q}(\mathbf{m}, A_n),$$

for $i \in I$, and $p \in P, q \in Q$. Since

$$\hat{\mathbf{m}}_{p,q}(E_i) \leq \mathbf{var}_{p,q}(\mathbf{m}, E_i), \quad i \in I, p \in P, q \in Q,$$

there is

$$\hat{\mathbf{m}}_{p,q}(E_i) \leq \mathbf{var}_{p,q}(\mathbf{m}, E) - \sum_{A_n \in l(i, E)} \mathbf{var}_{p,q}(\mathbf{m}, A_n),$$

where $i \in I, p \in P, q \in Q$. Since $\mathbf{var}_{p,q}(\mathbf{m}, E \cap \cdot) : \Sigma \rightarrow [0, \infty)$ is a finite real measure for every $p \in P, q \in Q$ with $q \mapsto_E p$, then for every $\varepsilon > 0, p \in P, q \in Q$ such that $q \mapsto_E p$, there exists an index $i_0 = i_0(\varepsilon, p, q, E) \in I$, such that

$$\hat{\mathbf{m}}_{p,q}(E_{i_0}) < \varepsilon \quad (3)$$

holds for every $i \geq i_0$, $i \in I$. Combining (1), (2), (3) and Definition 2.11 we see that the assertion is proved for the case when $E_i \in \Sigma$, $i \in I$, is a decreasing net of sets. The other cases of monotone nets of sets may be proved analogously.

Let now $G \subset T$ be an arbitrary set. Then there is exactly one (countable) set $F^* = \mathcal{A} \cap G$ with the property:

$$\hat{\mathbf{m}}_{p,q}^*(G) = \sup_{F \subset G, F \in \Sigma} \hat{\mathbf{m}}_{p,q}(F) = \hat{\mathbf{m}}_{p,q}(F^*), \quad p \in P, q \in Q.$$

The proof for the inner (p, q) -semivariation and the arbitrary net of subsets E_i , $i \in I$ goes by the same procedure as in the previous part of proof concerning the set system Σ . \square

Remark 3.2 Observe that Lemma 3.1 and Theorem 2.12 imply the statement of Theorem 2.7.

Theorem 3.3 (Egoroff) *Let $\mathbf{m} : \Sigma \rightarrow L(\mathbf{X}, \mathbf{Y})$ be a purely atomic measure, and let $E \in \Sigma$ be of finite (p, q) -variation of the measure \mathbf{m} . Let $\mathbf{f} : T \rightarrow \mathbf{X}$ be a measurable function, and $\mathbf{f}_i : T \rightarrow \mathbf{X}$, $i \in I$, be a net of measurable functions, such that*

$$\lim_{i \in I} p(\mathbf{f}_i(t) - \mathbf{f}(t)) = 0 \quad \text{for every } t \in E \text{ and } p \in P. \quad (4)$$

Then a net \mathbf{f}_i , $i \in I$, of functions \mathbf{m} -almost uniformly converges to \mathbf{f} on $E \in \Sigma$.

Proof. We have to prove that for a given $\varepsilon > 0$ there exist measurable sets $F = E(\varepsilon, p, q) \in \Sigma$, such that $\lim_{i \in I} \|\mathbf{f}_i - \mathbf{f}\|_{E \setminus F} = 0$, and $\hat{\mathbf{m}}_{p,q}(F) < \varepsilon$, for every $q \in Q$, $p \in P$, such that $q \rightarrow_E p$.

Suppose that (4) holds. For every $m \in \mathbb{N}$, $p \in P$, and $j \in I$, put

$$\begin{aligned} B_{m,j}^p &= E \cap \left\{ t \in T; p(\mathbf{f}_i(t) - \mathbf{f}(t)) < \frac{1}{m}, \quad i \geq j \right\} \\ &= E \cap \bigcap_{i \geq j} \left\{ t \in T; p(\mathbf{f}_i(t) - \mathbf{f}(t)) < \frac{1}{m}, \quad i \in I \right\}. \end{aligned}$$

Since there are countable many of atoms, $B_{m,j}^p \subset \Sigma$ and $\#B_{m,j}^p = \aleph_0$. Clearly, if $i, j \in I$ such that $i \leq j$, then $B_{m,i}^p \subset B_{m,j}^p$ for every $m \in \mathbb{N}$ and $p \in P$. Define $E_m^p = \bigcup_{j \in I} B_{m,j}^p$. The net $E_m^p \setminus B_{m,i}^p$, $i \in I$, clearly tends to void set for every $m \in \mathbb{N}$ and $p \in P$. Since \mathbf{m} is a purely atomic measure, then according to Lemma 3.1 its (p, q) -semivariation is GS-continuous, and therefore

$$\lim_{i \in I} \hat{\mathbf{m}}_{p,q}^*(E_m^p \setminus B_{m,i}^p) = 0,$$

for $q \in Q$, $p \in P$ such that $q \rightarrow_E p$.

Let $\varepsilon > 0$ be given. To every $p \in P$ and $m \in \mathbb{N}$ there exists an index $j = j(m, p) \in I$, such that for $q \rightarrow_E p$,

$$\hat{\mathbf{m}}_{p,q}(E_m^p \setminus B_{m,i}^p) < \varepsilon \cdot \alpha_p \cdot \beta_m,$$

holds for every $i \geq j(m, p)$, where $\{\alpha_p; p \in P\}$ is a summable system of positive numbers in the sense of Moor–Smith and $\{\beta_m; m \in \mathbb{N}\}$ is an absolutely convergent series of positive numbers. Put

$$F = \bigcup_{m \in \mathbb{N}} \bigcup_{p \in P} (E_m^p \setminus B_{m,j(m,p)}^p).$$

So, we have:

$$\begin{aligned}
\hat{\mathbf{m}}_{p,q}^*(F) &= \hat{\mathbf{m}}_{p,q}^* \left(\bigcup_{m \in \mathbb{N}} \bigcup_{p \in P} (E_m^p \setminus B_{m,j(m,p)}^p) \right) \\
&= \lim_{K=\{p_1, \dots, p_m\}} \hat{\mathbf{m}}_{p,q} \left(\sum_{m=1}^{\infty} \sum_{p \in K} (E_m^p \setminus B_{m,j(m,p)}^p) \right) \\
&\leq \lim_{K=\{p_1, \dots, p_m\}} \sum_{m=1}^{\infty} \sum_{p \in K} \hat{\mathbf{m}}_{p,q} (E_m^p \setminus B_{m,j(m,p)}^p) \\
&< \varepsilon.
\end{aligned}$$

Let us show that the convergence of net of functions \mathbf{f}_i , $i \in I$, is uniform on $E \setminus F$. Note that $\bigcup_{m=1}^{\infty} E_m^p = E$ for every $p \in P$. For a given $\eta > 0$ choose an $m_0 \in \mathbb{N}$, such that $\frac{1}{m} < \eta$. Then

$$\begin{aligned}
E \setminus F &= E \setminus \bigcup_{m \in \mathbb{N}} \bigcup_{p \in P} (E_m^p \setminus B_{m,j(m,p)}^p) \\
&= \bigcap_{m \in \mathbb{N}} \bigcap_{p \in P} B_{m,j(m,p)}^p \subset B_{m_0,j(m_0,p)}^p
\end{aligned}$$

for every $p \in P$. By definition of the set $B_{m_0,j(m_0,p)}^p$ we have that if $t \in B_{m_0,i}^p$, then

$$p(\mathbf{f}_i(t) - \mathbf{f}(t)) < \eta$$

for every $i \geq j(m_0, p)$. So, (4) implies that for every $\eta > 0$ and $p \in P$ there exists an index $j = j(\eta, p)$, such that for every $i \geq j(\eta, p)$, $i \in I$, there is

$$p(\mathbf{f}_i(t) - \mathbf{f}(t)) < \eta, \quad t \in E \setminus B_{m_0,i}^p \supset E \setminus F,$$

i.e., the net \mathbf{f}_i , $i \in I$, converges uniformly on $E \setminus F$. The proof is complete. \square

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